

Algebra 2 · Real and Complex Numbers · Worksheet Answer Key

Common Core N-RN.A.1, N-RN.A.2, N-CN.A.1, N-CN.A.2 · mathovore.fr

Every problem from the worksheet is solved step by step below. Final answers are in bold, and graph problems show the finished picture. Check each answer against your own work and look for the typical mistake: forgetting to replace i^2 with -1 .

Algebra 2 Common Core: N-RN.A.1 · N-RN.A.2 · N-CN.A.1 · N-CN.A.2

Problem 1: Classify numbers

Natural numbers start at 1. A number is rational if it can be written as a quotient of integers.

a) natural, integer, rational, real b) integer, rational, real c) rational, real d) irrational, real e) integer, rational, real (0 is not natural)

Problem 2: True or false

a) An integer n equals $\frac{n}{1}$. b) The real numbers include all irrational numbers. c) $\sqrt{49} = 7$, which is rational. d) $0.125 = \frac{1}{8}$. e) $\sqrt{2}$ is real but not rational.

a) True b) True c) False d) False e) False

Problem 3: Match points on a number line

Estimate: $\sqrt{5} \approx 2.24$ and $\sqrt{7} \approx 2.65$. The order from left to right is $-2.24, -0.5, 1.5, 2.65$.

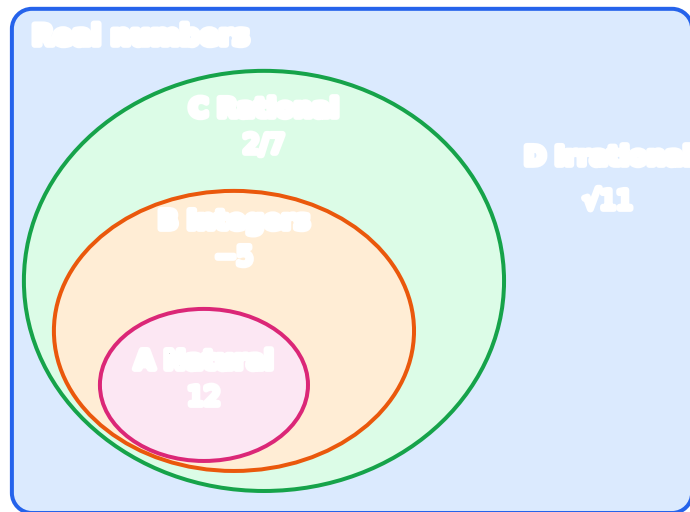
A = $-\sqrt{5}$, B = $-\frac{1}{2}$, C = $\frac{3}{2}$, D = $\sqrt{7}$. The rational numbers are $-\frac{1}{2}$ and $\frac{3}{2}$.

Problem 4: Regions of the number diagram

The innermost region holds the natural numbers, the next the integers, the next the rational numbers. The region outside the rational numbers, still inside the real numbers, holds the irrational numbers.

12 is natural; -5 is an integer but not natural; $\frac{2}{7}$ is rational but not an integer; $\sqrt{11}$ is irrational.

A natural, B integers, C rational, D irrational. 12 is in A, -5 in B, $\frac{2}{7}$ in C, $\sqrt{11}$ in D.



The four numbers placed in their smallest regions.

Problem 5: Square roots of negative numbers

Use $\sqrt{-a} = i\sqrt{a}$.

a) $\sqrt{-49} = 7i$. b) $\sqrt{-18} = i\sqrt{9 \cdot 2} = 3i\sqrt{2}$. c) $\sqrt{-75} = i\sqrt{25 \cdot 3} = 5i\sqrt{3}$. d) $-\sqrt{-81} = -9i$.

a) $7i$ b) $3i\sqrt{2}$ c) $5i\sqrt{3}$ d) $-9i$

Problem 6: Equations with imaginary solutions

a) $x = \pm\sqrt{-64} = \pm 8i$. b) $x^2 = -45$, so $x = \pm i\sqrt{45} = \pm 3i\sqrt{5}$. c) $2x^2 = -50$, $x^2 = -25$, $x = \pm 5i$.

a) $x = \pm 8i$ b) $x = \pm 3i\sqrt{5}$ c) $x = \pm 5i$

Problem 7: Real and imaginary parts

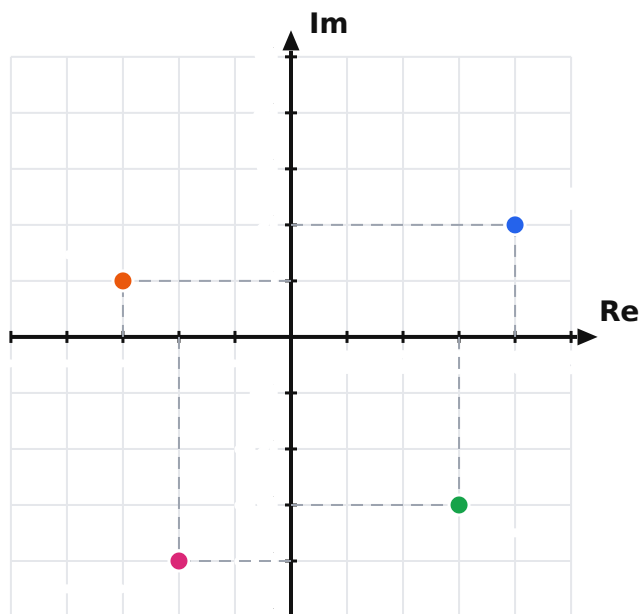
Write each number as $a+bi$. The imaginary part is the real number b , without the i .

a) 6 and 9 b) -4 and -7 c) 0 and 12 d) 15 and 0 e) $\frac{1}{2}$ and -1

Problem 8: Read points in the complex plane

Read the real part across the horizontal axis, then the imaginary part up or down.

P = $4+2i$, **Q** = $-3+i$, **R** = $-2-4i$, **S** = $3-3i$

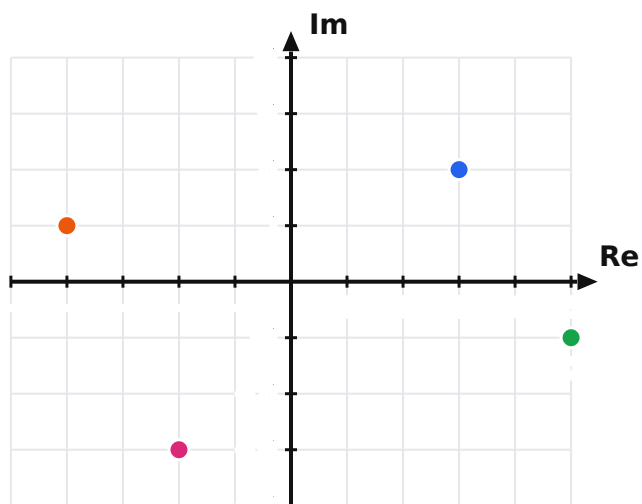


Reading across first, then up: $P = 4 + 2i$, $Q = -3 + i$, $R = -2 - 4i$, $S = 3 - 3i$.

Problem 9: Plot complex numbers

Move across the real axis by the real part and up or down by the imaginary part. A is 3 right and 2 up, B is 4 left and 1 up, C is 2 left and 3 down, D is 5 right and 1 down.

The points are plotted as shown.



$A = 3 + 2i$, $B = -4 + i$, $C = -2 - 3i$, $D = 5 - i$.

Problem 10: Add complex numbers

a) $3+2=5$ and $5i-8i=-3i$. b) $-4+6=2$ and $i+9i=10i$. c) $7-7=0$ and $-2i+2i=0$.

a) $5-3i$ b) $2+10i$ c) 0

Problem 11: Subtract complex numbers

Distribute the minus sign to both parts of the second number.

a) $9-3=6$, $4i-6i=-2i$. b) $2+3=5$, $-5i-i=-6i$. c) $-1+1=0$, $8i-3i=5i$.

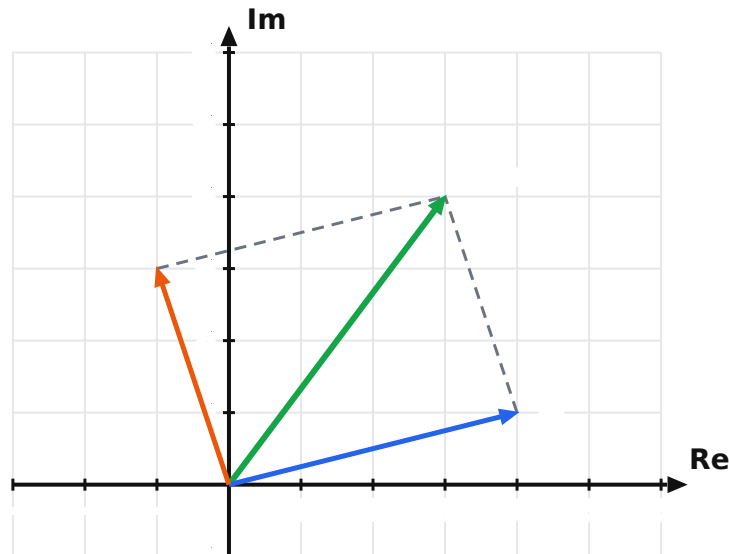
a) $6-2i$ b) $5-6i$ c) $5i$

Problem 12: Add arrows in the plane

The arrow z_1 ends at 4 across and 1 up. The arrow z_2 ends at 1 left and 3 up.

Sum: $(4+i)+(-1+3i)=3+4i$. Difference: $(4+i)-(-1+3i)=5-2i$.

$z_1=4+i$, $z_2=-1+3i$, $z_1+z_2=3+4i$, $z_1-z_2=5-2i$



The sum is the diagonal of the parallelogram.

Problem 13: Multiply by an imaginary number

a) $6i+15i^2=6i-15$. b) $-8i+6i^2=-8i-6$. c) $7i-i^2=7i+1$.

a) $-15+6i$ b) $-6-8i$ c) $1+7i$

Problem 14: Multiply two binomials

a) $3+i+6i+2i^2=3+7i-2$. b) $5+20i-2i-8i^2=5+18i+8$. c) $9-3i-3i+i^2=9-6i-1$.

a) $1+7i$ b) $13+18i$ c) $8-6i$

Problem 15: Multiply conjugates

The middle terms cancel, leaving the real number a^2+b^2 .

a) $16-9i^2=16+9=25$. b) $36-i^2=37$. c) $1-3i^2=1+3=4$.

a) 25 b) 37 c) 4

Problem 16: Powers of i

Divide each exponent by 4 and use the remainder.

a. $6 = 4(1) + 2$, so $i^6 = i^2 = -1$.

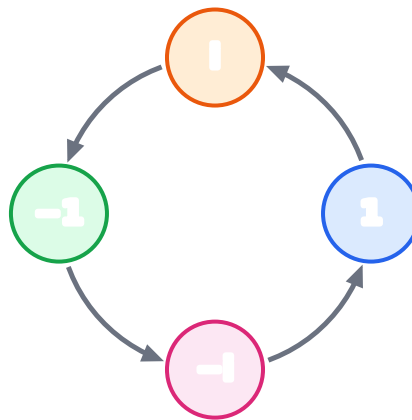
b. $13 = 4(3) + 1$, so $i^{13} = i$.

c. $30 = 4(7) + 2$, so $i^{30} = -1$.

d. $47 = 4(11) + 3$, so $i^{47} = -i$.

e. $64 = 4(16) + 0$, so $i^{64} = 1$.

a) -1 b) i c) -1 d) $-i$ e) 1



Remainder 0 gives 1, 1 gives i, 2 gives -1 , 3 gives $-i$.

Problem 17: Simplify with powers of i

a) $2(-i)+5(-1)-1+6=-2i-5-1+6$, which is $-2i$. b) $9i^2=-9$. c) $(-2)^3i^3=-8(-i)=8i$.

a) $-2i$ b) -9 c) $8i$

Problem 18: Verify a solution

Substitute: $(1+2i)^2=1+4i+4i^2=-3+4i$. Then $-2(1+2i)=-2-4i$.

Add: $(-3+4i)+(-2-4i)+5=0$.

The left side equals 0, so $1+2i$ is a solution.

Problem 19: Impedance in a circuit

Add real parts: $3+2=5$. Add imaginary parts: $4i-i=3i$.

$5+3i$ ohms

Problem 20: Voltage across a circuit

$(2+i)(3-2i)=6-4i+3i-2i^2=6-i+2$.

$8-i$ volts

Problem 21: Find the error

The rule $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$ does not work when both numbers under the roots are negative. Noah must rewrite each root with i first.

$\sqrt{-4}=2i$ and $\sqrt{-9}=3i$, so $2i\cdot 3i=6i^2=-6$.

The correct value is -6 .

Problem 22: Moving on a game map

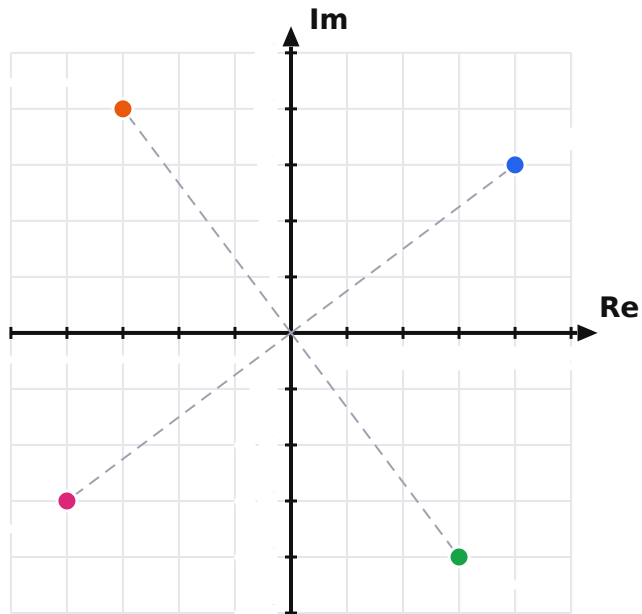
After the first move: $(3+2i)+(-5+4i)=-2+6i$. After the second move: $(-2+6i)+(2-7i)=0-i$.

The character ends at $-i$, which is the point $(0,-1)$.

Problem 23: Rotating a drone

One time: $i(4+3i)=4i+3i^2=-3+4i$. Two times: $i(-3+4i)=-3i+4i^2=-4-3i$. Three times: $i(-4-3i)=-4i-3i^2=3-4i$.

$-3+4i$, $-4-3i$, $3-4i$. Each multiplication by i turns the point a quarter turn (90°) counterclockwise about the origin.



Each multiplication by i turns the point a quarter turn counterclockwise about the origin.

Problem 24: Make a product real

Expand: $2k - ki + 6i - 3i^2 = (2k + 3) + (6 - k)i$. The product is real when the imaginary part is zero: $6 - k = 0$, so $k = 6$.

Then the real part is $2(6) + 3 = 15$.

$k = 6$, and the product is 15.