

Algebra 2 · Real and Complex Numbers · Lesson

Common Core N-RN.A.1, N-RN.A.2, N-CN.A.1, N-CN.A.2 · mathovore.fr

A complex number has the form $a + bi$, where a and b are real numbers and i is the imaginary unit with $i^2 = -1$. Every real number is complex (take $b = 0$), so equations like $x^2 = -9$ now have solutions. Add and subtract the parts separately, multiply by distributing, and replace i^2 with -1 .

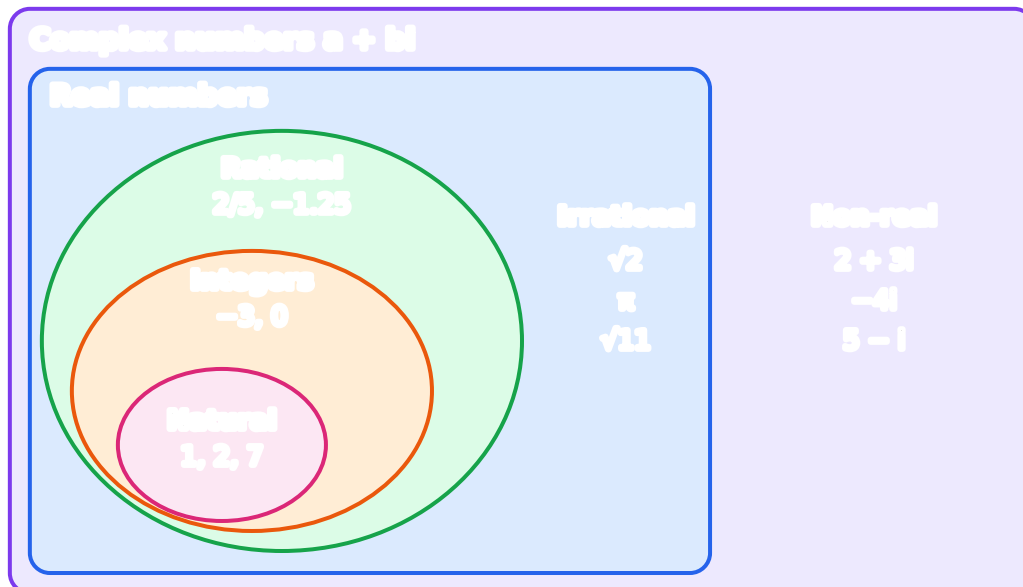
Algebra 2 Common Core: N-RN.A.1 · N-RN.A.2 · N-CN.A.1 · N-CN.A.2

Key facts:

- A complex number is $a + bi$ with real a and b , and $i^2 = -1$.
- Every real number is a complex number with $b = 0$.
- Add or subtract real parts and imaginary parts separately.
- To multiply, distribute and then replace i^2 by -1 .
- The powers of i cycle through i , -1 , $-i$, 1 every four exponents.

What sets of numbers make up the real numbers?

The **natural numbers** are 1, 2, 3 and so on. The **integers** add 0 and the negatives. A **rational number** is a quotient $\frac{a}{b}$ of integers with $b \neq 0$; its decimal stops or repeats. An **irrational number** such as $\sqrt{2}$ or π has a decimal that never stops or repeats. Together they fill the number line: the **real numbers**.



Every natural number is an integer, every integer is rational, and the rational and irrational numbers together make the real numbers.

What is the imaginary unit i ?

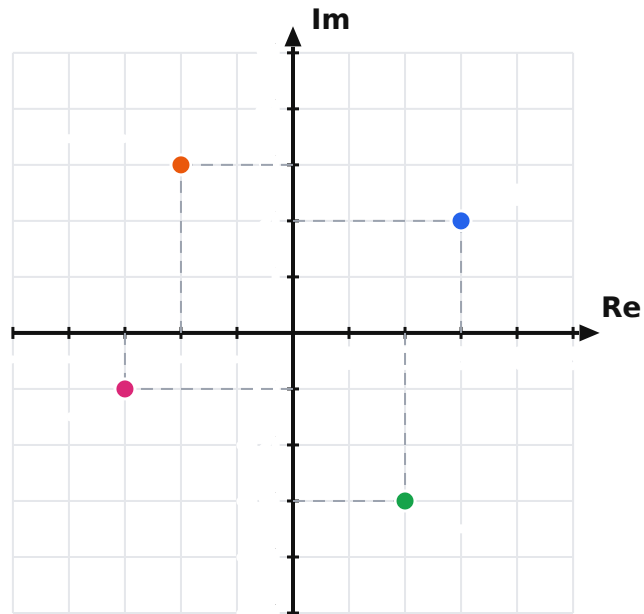
No real number squared gives a negative result, so mathematicians define a new number.

Imaginary unit

$i^2 = -1$, so $i = \sqrt{-1}$. For any $a > 0$, $\sqrt{-a} = i\sqrt{a}$. Example: $\sqrt{-36} = 6i$ and $\sqrt{-20} = 2i\sqrt{5}$.

What is a complex number?

A **complex number** has the form $a+bi$, where a and b are real. The number a is the **real part** and b is the **imaginary part**. If $b=0$ the number is real, so every real number is also complex. Two complex numbers are equal only when both parts match. In the complex plane the point (a,b) stands for $a+bi$.

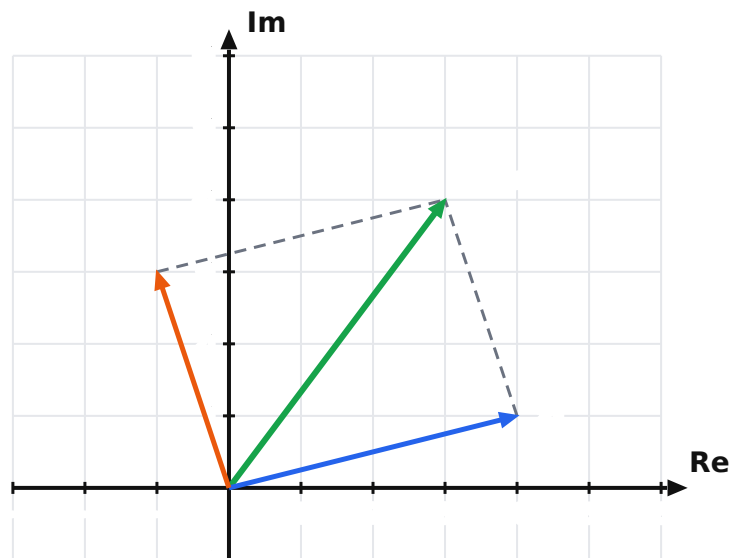


Each complex number $a + bi$ is the point (a, b) : real part across, imaginary part up.

How do you add, subtract and multiply complex numbers?

To add or subtract, combine real parts and imaginary parts separately:

$(a+bi)+(c+di)=(a+c)+(b+d)i$. To multiply, use the distributive property and replace i^2 with -1 .



$(4 + i) + (-1 + 3i) = 3 + 4i$: adding complex numbers is adding arrows.

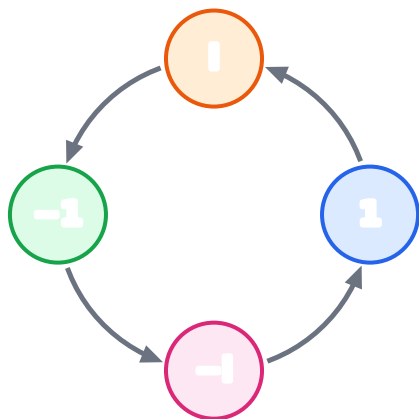
Worked example: find $(2+3i)(4-5i)$.

1. Distribute: $2 \cdot 4 + 2 \cdot (-5i) + 3i \cdot 4 + 3i \cdot (-5i)$.

2. Simplify: $8-10i+12i-15i^2$.
3. Replace i^2 with -1 : $8-10i+12i+15$.
4. Combine like terms: $23+2i$.

What are the powers of i ?

The powers of i repeat every four steps: $i^1=i$, $i^2=-1$, $i^3=-i$, $i^4=1$. Divide the exponent by 4 and use the remainder: 0 gives 1, 1 gives i , 2 gives -1 , 3 gives $-i$. For i^{27} , the remainder of $27 \div 4$ is 3, so $i^{27} = -i$.



Multiplying by i moves one step around the cycle.

Real and Complex Numbers: frequently asked questions

Is zero a complex number?

Yes. Zero is $0+0i$, so it is real, rational, an integer, and complex. It is not a natural number in this course.

Why is $\sqrt{-a} = i\sqrt{a}$ only for positive a ?

The rule rewrites the square root of a negative number using the imaginary unit. Writing $\sqrt{-16} = i\sqrt{16} = 4i$ keeps the number organized before you multiply.

What is the difference between an imaginary number and a complex number?

A complex number is any $a+bi$. It is called imaginary when $b \neq 0$ and purely imaginary when $a=0$, such as $5i$.

How do you find a large power of i ?

Divide the exponent by 4 and keep only the remainder. Since $i^4=1$, the multiples of 4 drop

out. For i^{50} the remainder is 2, so the answer is -1 .

Can the sum of two complex numbers be real?

Yes. When the imaginary parts are opposites, they cancel. For example $(3+5i)+(2-5i)=5$.