

Algebra 2 · Rational Functions and Their Graphs · Lesson

Common Core F-IF.C.7d, F-IF.B.4 · mathovore.fr

A rational function is a quotient of two polynomials, such as $f(x) = 2/(x - 3) + 1$. Its graph can have vertical asymptotes where the denominator is zero, a horizontal asymptote that shows end behavior, and holes where factors cancel. Inverse variation, $y = k/x$, is the simplest model.

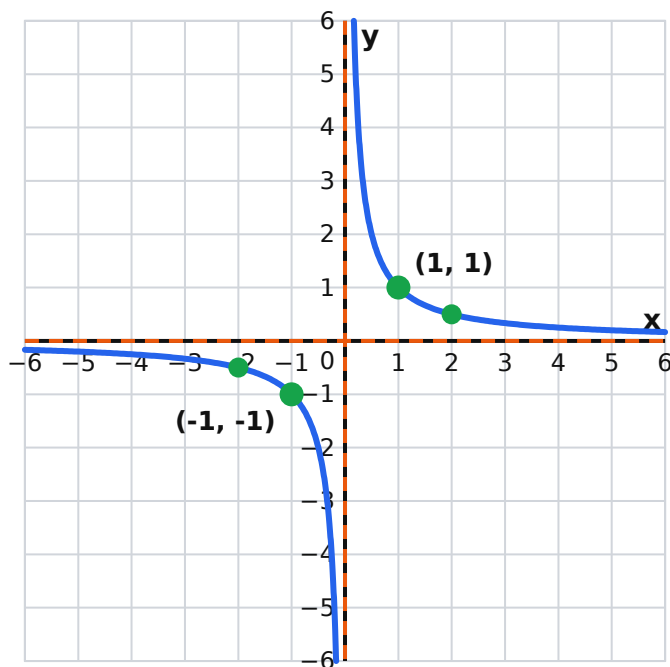
Algebra 2 Common Core: F-IF.C.7d · F-IF.B.4

Key facts:

- Standards F-IF.C.7d and F-IF.B.4: graph rational functions and identify asymptotes, intercepts, domain and end behavior.
- The graph of $y = \frac{a}{x-h} + k$ has asymptotes $x = h$ and $y = k$ and center (h, k) .
- A canceled factor $x - c$ leaves a hole at $x = c$; a factor that remains in the denominator gives a vertical asymptote.
- Compare degrees: smaller numerator gives $y = 0$, equal degrees give the ratio of leading coefficients, larger numerator gives none.
- Inverse variation $y = \frac{k}{x}$ means the product xy is constant.

What is a rational function?

A **rational function** is a quotient of two polynomials, $f(x) = \frac{p(x)}{q(x)}$, where $q(x) \neq 0$. The simplest one is $y = \frac{1}{x}$. Its domain is every real number except the zeros of the denominator, and its graph is a **hyperbola** with two separate branches.



The parent rational function $y = \frac{1}{x}$ has two branches and the dashed asymptotes $x = 0$ and $y = 0$.

How do you find vertical and horizontal asymptotes?

Vocabulary

Vertical asymptote: a line $x = c$ that the graph approaches but never touches. It occurs where the denominator is zero after common factors are removed.

Horizontal asymptote: a line $y = L$ that the graph approaches as x gets very large or very negative.

Hole: a single missing point that comes from a factor canceled in the numerator and denominator.

Compare the degrees of the numerator and denominator to find the horizontal asymptote.

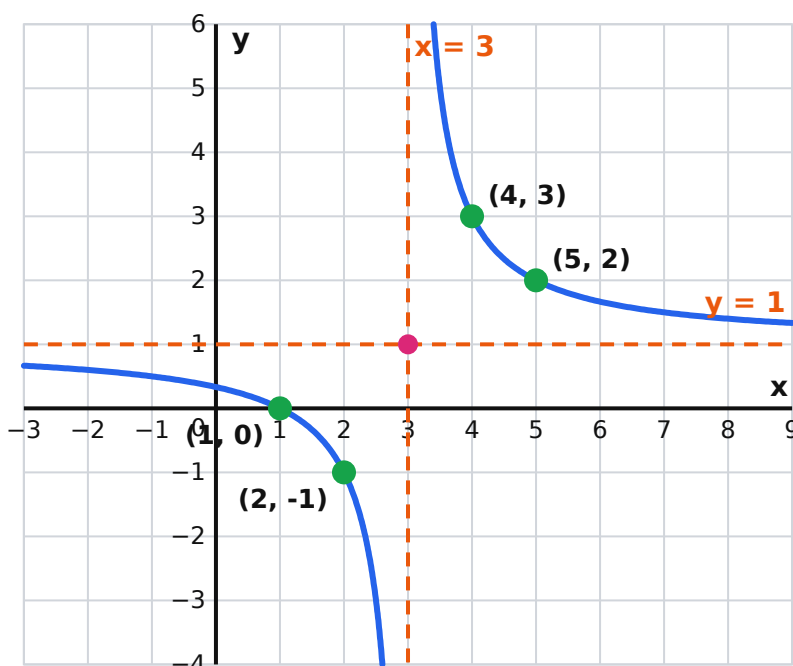
Degrees	Horizontal asymptote	Example
Numerator smaller	$y = 0$	$\frac{3}{x^2+1}$ gives $y = 0$
Degrees equal	ratio of leading coefficients	$\frac{6x+1}{2x-5}$ gives $y = 3$
Numerator larger	none	$\frac{x^2+1}{x-1}$ has none

How do you graph $y = a/(x - h) + k$?

This form is $y = \frac{1}{x}$ stretched by a , moved right h and up k . The asymptotes are $x = h$ and $y = k$, and they cross at the center (h, k) . If $a > 0$ the branches lie up-right and down-left of the center; if $a < 0$ they are reversed.

1. Draw dashed lines $x = h$ and $y = k$.
2. Make a table with x -values on both sides of h .
3. Plot the points and sketch each branch toward the asymptotes.

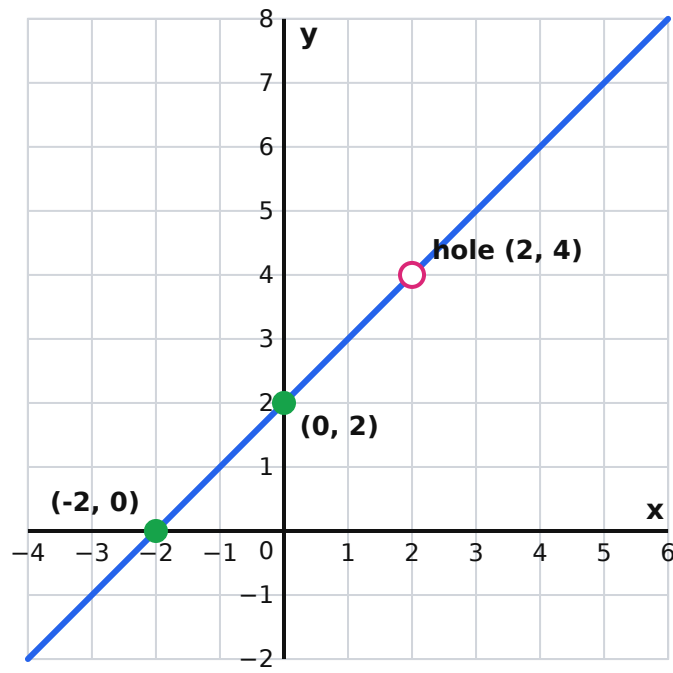
Worked example. Graph $y = \frac{2}{x-3} + 1$. The asymptotes are $x = 3$ and $y = 1$. Then $x = 4$ gives 3, $x = 5$ gives 2, $x = 2$ gives -1 and $x = 1$ gives 0.



Graph of $y = \frac{2}{x-3} + 1$. The asymptotes cross at the center $(3, 1)$, and the branches sit in the upper right and lower left.

What is a hole in a graph?

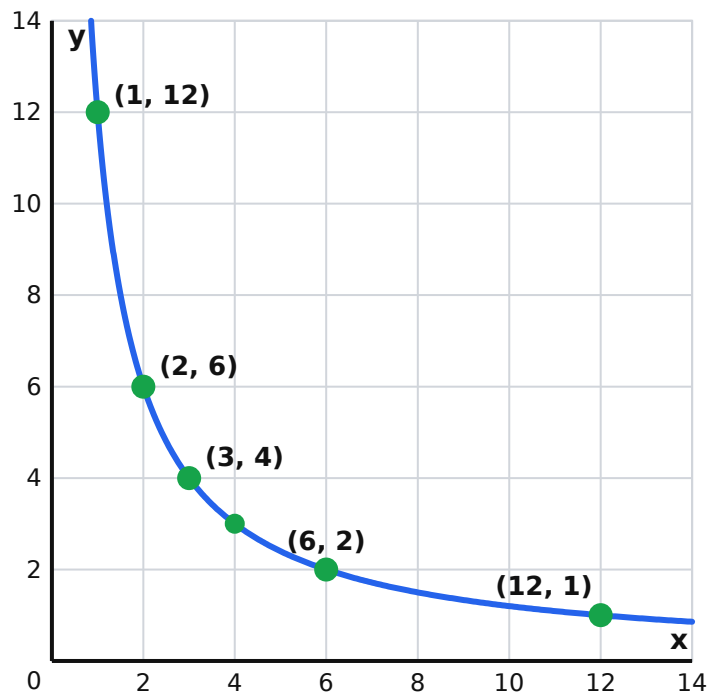
Factor first. For $f(x) = \frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2}$, the factor $x-2$ cancels, so the graph is the line $y = x+2$ except at $x = 2$. The hole is at $(2, 4)$, and there is no vertical asymptote there.



The graph of $f(x) = \frac{x^2 - 4}{x - 2}$ is the line $y = x + 2$ with a hole at $(2, 4)$.

What is inverse variation?

Two quantities vary inversely when $y = \frac{k}{x}$, or $xy = k$, for a constant $k \neq 0$. If $y = 3$ when $x = 4$, then $k = 12$, and $y = \frac{12}{x}$ gives $y = 2$ when $x = 6$.



Inverse variation $y = \frac{12}{x}$: every point has product $xy = 12$, so doubling x cuts y in half.

Rational Functions and Their Graphs: frequently asked questions

How do you find the vertical asymptote of a rational function?

Factor the numerator and denominator and cancel common factors. Set each remaining factor of the denominator equal to zero. For $y = \frac{5}{x+7}$ the vertical asymptote is $x = -7$.

What is the difference between a hole and a vertical asymptote?

Both occur where the denominator is zero. A hole comes from a factor that cancels, so only one point is missing. An asymptote comes from a factor that stays in the denominator, so the graph rises or falls without bound.

How do you find a horizontal asymptote?

Compare the degrees of the numerator and denominator. If the numerator has the smaller degree, the asymptote is $y = 0$. If the degrees are equal, divide the leading coefficients, so $\frac{6x+1}{2x-5}$ has $y = 3$.

What is the domain of a rational function?

The domain is all real numbers except the values that make the denominator zero in the original expression, including values that create a hole. For $\frac{x+1}{x^2-9}$ the domain excludes $x = 3$ and $x = -3$.

How do you know if a situation is inverse variation?

Multiply paired values. If xy gives the same number every time, the relationship is $y = \frac{k}{x}$. A trip of 240 miles has time $t = \frac{240}{r}$, because speed times time is always 240.