

Algebra 2 · Rational Expressions and Equations

· Worksheet Answer Key

Common Core A-APR.D.6, A-APR.D.7, A-REI.A.2 · mathovore.fr

This answer key gives a full worked solution for each of the 22 problems, with excluded values stated and every solution of a rational equation checked. Compare your factoring and your final bold answer with these solutions.

Algebra 2 Common Core: A-APR.D.6 · A-APR.D.7 · A-REI.A.2

Problem 1: Find the excluded values

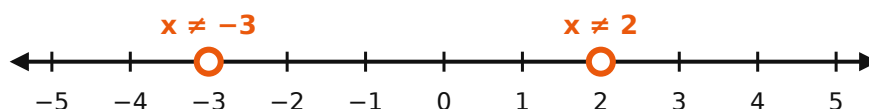
Set each denominator equal to 0.

a) $x - 4 = 0$, so $x = 4$.

b) $x^2 - 9 = (x - 3)(x + 3) = 0$, so $x = 3$ or $x = -3$.

c) $x^2 + x - 6 = (x + 3)(x - 2) = 0$, so $x = -3$ or $x = 2$.

a) $x \neq 4$ b) $x \neq \pm 3$ c) $x \neq -3$ and $x \neq 2$



The excluded values for part c are -3 and 2 .

Problem 2: Simplify monomial quotients

a) Divide by the GCF $6x^2$: $\frac{18x^4}{24x^2} = \frac{3x^2}{4}$.

b) Divide by the GCF $5xy$: $\frac{20x^3y}{35xy^2} = \frac{4x^2}{7y}$.

a) $\frac{3x^2}{4}$ b) $\frac{4x^2}{7y}$

Problem 3: Simplify by factoring

a) $\frac{(x-3)(x+3)}{(x+3)(x-2)} = \frac{x-3}{x-2}$. The original denominator is $(x+3)(x-2)$, so $x \neq -3$ and $x \neq 2$.

b) $\frac{(x+2)(x+3)}{(x+1)(x+2)} = \frac{x+3}{x+1}$. The original denominator is $(x+1)(x+2)$, so $x \neq -1$ and $x \neq -2$.

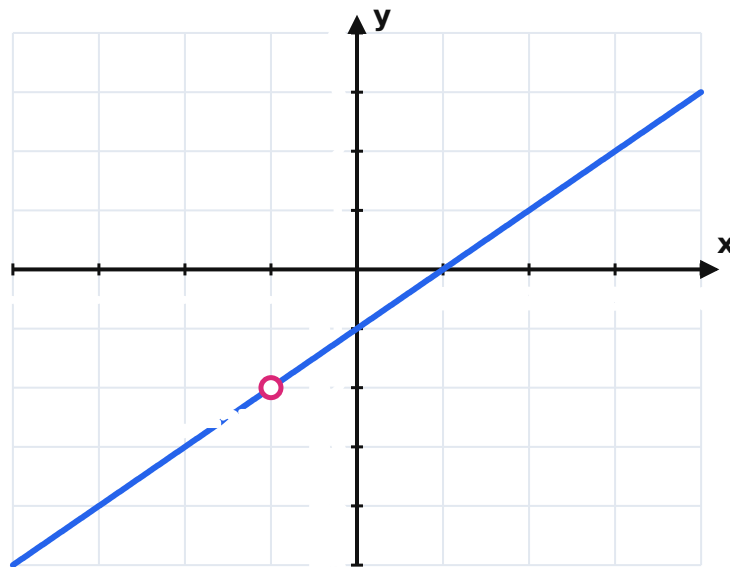
a) $\frac{x-3}{x-2}$, $x \neq -3$, 2 b) $\frac{x+3}{x+1}$, $x \neq -2$, -1

Problem 4: Why the graph has a hole

Factor: $\frac{(x-1)(x+1)}{x+1} = x-1$ for $x \neq -1$. The factor $x+1$ cancels, but the original expression is undefined at $x = -1$, so the line $y = x-1$ is missing one point.

At $x = -1$ the line has $y = -1-1 = -2$.

Simplified: $x-1$. **The hole is at** $(-1, -2)$.



The line $y = x-1$ with a hole at $(-1, -2)$.

Problem 5: Opposite factors

a) $4-x = -(x-4)$, so $\frac{4-x}{x-4} = -1$ for $x \neq 4$.

b) $9-x^2 = (3-x)(3+x) = -(x-3)(x+3)$, so the expression equals $-(x+3)$ for $x \neq 3$.

a) -1 , $x \neq 4$ b) $-(x+3) = -x-3$, $x \neq 3$

Problem 6: Multiply rational expressions

a) $\frac{(x-2)(x+2)}{x+5} \cdot \frac{2(x+5)}{x-2} = 2(x+2)$, with $x \neq -5$, 2.

b) $\frac{x(x+3)}{(x-1)(x+1)} \cdot \frac{x-1}{x} = \frac{x+3}{x+1}$, with $x \neq 0$, 1, -1 .

a) $2(x+2)$ b) $\frac{x+3}{x+1}$

Problem 7: Divide rational expressions

a) Multiply by the reciprocal: $\frac{(x-3)(x+2)}{x+4} \cdot \frac{(x-4)(x+4)}{x-3} = (x+2)(x-4)$. Excluded: $x \neq -4, 3, 4$.

b) $\frac{6x^2}{5y} \cdot \frac{10y^2}{3x} = \frac{60x^2y^2}{15xy} = 4xy$, with $x \neq 0, y \neq 0$.

a) $(x+2)(x-4)$ b) $4xy$

Problem 8: Same denominator

a) $\frac{(3x+1)+(2x-5)}{x+4} = \frac{5x-4}{x+4}$.

b) Distribute the minus sign: $\frac{5x+2-2x+7}{x-1} = \frac{3x+9}{x-1} = \frac{3(x+3)}{x-1}$.

a) $\frac{5x-4}{x+4}$ b) $\frac{3(x+3)}{x-1}$

Problem 9: Different denominators

a) LCD $x(x+1)$: $\frac{2(x+1)+3x}{x(x+1)} = \frac{5x+2}{x(x+1)}$.

b) LCD $12x^2$: $\frac{5 \cdot 2x}{12x^2} - \frac{1 \cdot 3}{12x^2} = \frac{10x-3}{12x^2}$.

a) $\frac{5x+2}{x(x+1)}$ b) $\frac{10x-3}{12x^2}$

Problem 10: Binomial denominators

The LCD is $(x-2)(x+3)$. Then $\frac{4(x+3)-3(x-2)}{(x-2)(x+3)} = \frac{4x+12-3x+6}{(x-2)(x+3)} = \frac{x+18}{(x-2)(x+3)}$.

$\frac{x+18}{(x-2)(x+3)}$, $x \neq 2, -3$

Problem 11: Factor the denominators first

a) $x^2-9=(x-3)(x+3)$ is the LCD. $\frac{x-(x+3)}{(x-3)(x+3)} = \frac{-3}{(x-3)(x+3)}$.

b) $x^2-x-6=(x-3)(x+2)$ is the LCD. $\frac{2+(x-3)}{(x-3)(x+2)} = \frac{x-1}{(x-3)(x+2)}$.

a) $\frac{-3}{(x-3)(x+3)}$ b) $\frac{x-1}{(x-3)(x+2)}$

Problem 12: Rewrite using long division

a) $x^2 \div x = x$; subtract x^2+x to get $2x+5$; $2x \div x = 2$; subtract $2x+2$ to get 3. So $x+2+\frac{3}{x+1}$.

b) $2x^2 \div x = 2x$; subtract $2x^2 - 4x$ to get $3x - 7$; $3x \div x = 3$; subtract $3x - 6$ to get -1 . So $2x + 3 - \frac{1}{x-2}$.

a) $x + 2 + \frac{3}{x+1}$ b) $2x + 3 - \frac{1}{x-2}$

Problem 13: Rewrite by inspection

a) $2x + 7 = 2(x + 3) + 1$, so $\frac{2x+7}{x+3} = 2 + \frac{1}{x+3}$.

b) $3x - 4 = 3(x - 1) - 1$, so $\frac{3x-4}{x-1} = 3 - \frac{1}{x-1}$.

a) $2 + \frac{1}{x+3}$ b) $3 - \frac{1}{x-1}$

Problem 14: Solve a proportion-style equation

a) Cross multiply: $3(x+4) = 5x$, so $3x + 12 = 5x$ and $x = 6$. Check: $\frac{3}{6} = \frac{1}{2}$ and $\frac{5}{10} = \frac{1}{2}$.

b) Cross multiply: $2(x+3) = 6(x-1)$, so $2x + 6 = 6x - 6$ and $x = 3$. Check: $\frac{2}{3} = 1$ and $\frac{6}{6} = 1$.

a) $x = 6$ b) $x = 3$

Problem 15: Rational equation with a quadratic

The LCD is $2x(x+3)$. Multiply every term: $2(x+3) + 2x = x(x+3)$, so $4x + 6 = x^2 + 3x$.

Then $x^2 - x - 6 = 0$, so $(x-3)(x+2) = 0$ and $x = 3$ or $x = -2$.

Check $x = 3$: $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$. Check $x = -2$: $-\frac{1}{2} + 1 = \frac{1}{2}$.

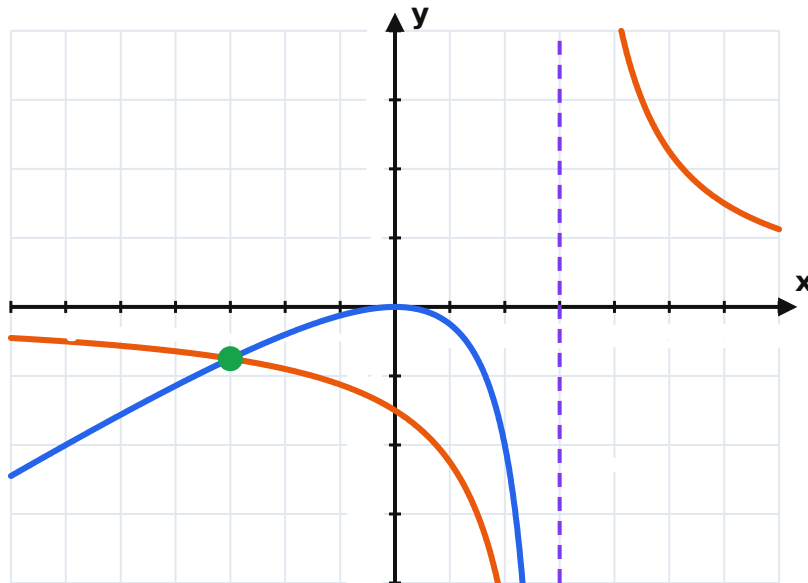
$x = 3$ or $x = -2$

Problem 16: Spot the extraneous solution

Multiply both sides by $x-3$: $x^2 = 9$, so $x = 3$ or $x = -3$.

At $x = 3$ both denominators are 0, so $x = 3$ is not allowed. At $x = -3$: $\frac{9}{-6} = -\frac{3}{2}$ and $\frac{9}{-6} = -\frac{3}{2}$, so it works.

$x = -3$ is the only solution; $x = 3$ is extraneous. The graphs meet only at $(-3, -1.5)$.



The curves meet only at $(-3, -1.5)$; $x=3$ is an asymptote, not a solution.

Problem 17: Equation with no solution

Multiply by $x-4$: $x = 4 + 2(x-4)$, so $x = 2x - 4$ and $x = 4$.

But $x = 4$ makes the denominator 0, so it is extraneous.

No solution.

Problem 18: Average cost

a) Solve $\frac{120+8n}{n} = 11$. Multiply by n : $120+8n = 11n$, so $3n = 120$ and $n = 40$. Check:
 $\frac{120+320}{40} = \frac{440}{40} = 11$.

b) $\frac{120+8n}{n} = \frac{120}{n} + 8$. As n grows, $\frac{120}{n}$ gets close to 0, so $A(n)$ approaches 8.

a) 40 hoodies b) \$8 per hoodie

Problem 19: Painting a fence together

Rates: Maya $\frac{1}{6}$ fence per hour, Jamal $\frac{1}{3}$. Together: $\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$.

Multiply by $6t$: $t + 2t = 6$, so $3t = 6$ and $t = 2$. Check: in 2 hours Maya paints $\frac{2}{6} = \frac{1}{3}$ and Jamal paints $\frac{2}{3}$, a total of 1 fence.

2 hours



Together, in 2 hours: Maya $\frac{1}{3}$, Jamal $\frac{2}{3}$.

Problem 20: Fill pipe and drain

The pipe adds $\frac{1}{4}$ of the tank per hour and the drain removes $\frac{1}{12}$. So $\frac{1}{4} - \frac{1}{12} = \frac{1}{6}$.

Multiply by 12t: $3t - t = 12$, so $2t = 12$ and $t = 6$.

6 hours

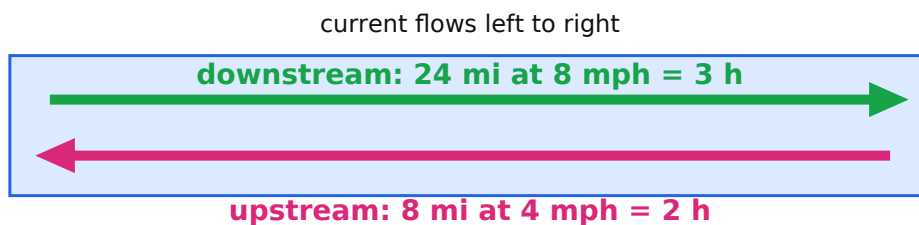
Problem 21: Boat on a river

Time = distance $\div$ speed. The equation is $\frac{24}{r+2} + \frac{8}{r-2} = 5$.

Multiply by $(r+2)(r-2)$: $24(r-2) + 8(r+2) = 5(r^2-4)$, so $32r - 32 = 5r^2 - 20$. Then $5r^2 - 32r + 12 = 0$, which factors as $(5r-2)(r-6) = 0$.

So $r = \frac{2}{5}$ or $r = 6$. Upstream speed is $r-2$, which must be positive, so $r = \frac{2}{5}$ is rejected. Check $r = 6$: $\frac{24}{8} + \frac{8}{4} = 3 + 2 = 5$.

6 mph



With $r = 6$ mph: 3 hours plus 2 hours is 5 hours.

Problem 22: Decorating the gym

Let a be Aisha's time alone. Then $\frac{1}{6} + \frac{1}{a} = \frac{1}{4}$.

Multiply by $12a$: $2a + 12 = 3a$, so $a = 12$. Check: $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$.

12 hours