

Algebra 2 · Radicals and Rational Exponents · Worksheet Answer Key

Common Core N-RN.A.1, N-RN.A.2, A-REI.A.2 · mathovore.fr

Each solution shows the rewrite, the simplification and the check, with the final answer in bold. When you solve a radical equation, substitute every candidate into the original equation before you accept it.

Algebra 2 Common Core: N-RN.A.1 · N-RN.A.2 · A-REI.A.2

Problem 1: Evaluate nth roots

- a. $13^2 = 169$, so 13.
- b. $(-4)^3 = -64$, so -4.
- c. $2^4 = 16$, so 2.
- d. $(-2)^5 = -32$, so -2.

Problem 2: Real or not real

- a. An even root of a negative number: **not a real number**.
- b. $(-7)^3 = -343$, so -7.
- c. Even index, negative radicand: **not a real number**.
- d. $(-1)^5 = -1$, so -1.

Problem 3: Radical to exponent form

- a. $x^{1/3}$
- b. The index 5 is the denominator: $y^{2/5}$.
- c. $z^{3/4}$
- d. $7^{3/2}$

Problem 4: Exponent to radical form

- a. $\sqrt{5}$
- b. $\sqrt[3]{x^2}$
- c. $\sqrt[4]{(3a)^3}$
- d. Only m carries the exponent, so the factor 2 stays outside: $2\sqrt[3]{m}$.

Problem 5: Evaluate rational exponents

- a. $\sqrt{25} = 5$.

- b. $\sqrt[3]{64} = 4$.
- c. $(\sqrt[4]{16})^3 = 2^3 = 8$.
- d. $(\sqrt[3]{8})^2 = 2^2 = 4$.
- e. $(\sqrt[5]{32})^3 = 2^3 = 8$.

Problem 6: Negative rational exponents

- a. $\frac{1}{9^{1/2}} = \frac{1}{3} \cdot \frac{1}{3}$
- b. $27^{2/3} = 3^2 = 9$, so $\frac{1}{9}$.
- c. $16^{3/4} = 2^3 = 8$, so $\frac{1}{8}$.
- d. Flip the base: $4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$.

Problem 7: Simplify square roots

- a. $\sqrt{16 \cdot 3} = 4\sqrt{3}$. $4\sqrt{3}$
- b. $\sqrt{9 \cdot 10} = 3\sqrt{10}$.
- c. $\sqrt{100 \cdot 2} = 10\sqrt{2}$.
- d. $\sqrt{25 \cdot 3} = 5\sqrt{3}$.

Problem 8: Simplify higher roots

- a. $\sqrt[3]{8 \cdot 3} = 2\sqrt[3]{3}$.
- b. $\sqrt[3]{125 \cdot 2} = 5\sqrt[3]{2}$.
- c. $\sqrt[3]{-27 \cdot 2} = -3\sqrt[3]{2}$.
- d. $\sqrt[4]{16 \cdot 3} = 2\sqrt[4]{3}$.

Problem 9: Radicals with variables

- a. $\sqrt{9x^4 \cdot 2x} = 3x^2\sqrt{2x}$.
- b. $\sqrt[3]{8y^6 \cdot 2y} = 2y^2\sqrt[3]{2y}$.
- c. $\sqrt{25a^2b^4 \cdot 2a} = 5ab^2\sqrt{2a}$.
- d. $\sqrt[3]{27m^3n^9 \cdot 2m^2} = 3mn^3\sqrt[3]{2m^2}$.

Problem 10: Multiply radicals

- a. $\sqrt{60} = \sqrt{4 \cdot 15} = 2\sqrt{15}$.
- b. $\sqrt[3]{24} = 2\sqrt[3]{3}$.
- c. $15\sqrt{28} = 15 \cdot 2\sqrt{7} = 30\sqrt{7}$.
- d. $2\sqrt{18} = 2 \cdot 3\sqrt{2} = 6\sqrt{2}$.

Problem 11: Add and subtract radicals

- a. $6\sqrt{3} + 3\sqrt{3} = 9\sqrt{3}$.
- b. $10\sqrt{2} - 6\sqrt{2} = 4\sqrt{2}$.

- c. $2\sqrt[3]{2} + 3\sqrt[3]{2} = 5\sqrt[3]{2}$.
 d. $2\sqrt{5} - 3\sqrt{5} + \sqrt{5} = 0$.

Problem 12: Rationalize denominators

- a. $\frac{6\sqrt{3}}{3} = 2\sqrt{3}$.
 b. $\frac{5\sqrt{10}}{10} = \frac{\sqrt{10}}{2}$.
 c. Multiply by $\frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$: the denominator becomes $7-3=4$, so the result is $\frac{\sqrt{7}+\sqrt{3}}{4}$.
 d. Multiply by $\frac{\sqrt[3]{4}}{\sqrt[3]{4}}$: $\frac{3\sqrt[3]{4}}{\sqrt[3]{8}} = \frac{3\sqrt[3]{4}}{2}$.

Problem 13: Exponent properties

- a. $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$: $x^{5/6}$.
 b. $\frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$: $y^{1/2}$.
 c. $\frac{5}{4} - \frac{1}{4} = 1$: x .
 d. $8^{2/3} a^2 = 4a^2$.

Problem 14: Change the index

- a. $x^{4/6} = x^{2/3} = \sqrt[3]{x^2}$.
 b. $9^{1/4} = (3^2)^{1/4} = 3^{1/2} = \sqrt{3}$.
 c. $64^{1/6} = 2$, since $2^6 = 64$.
 d. $x^{12/3} = x^4$, then $(x^4)^{1/2} = x^2$.

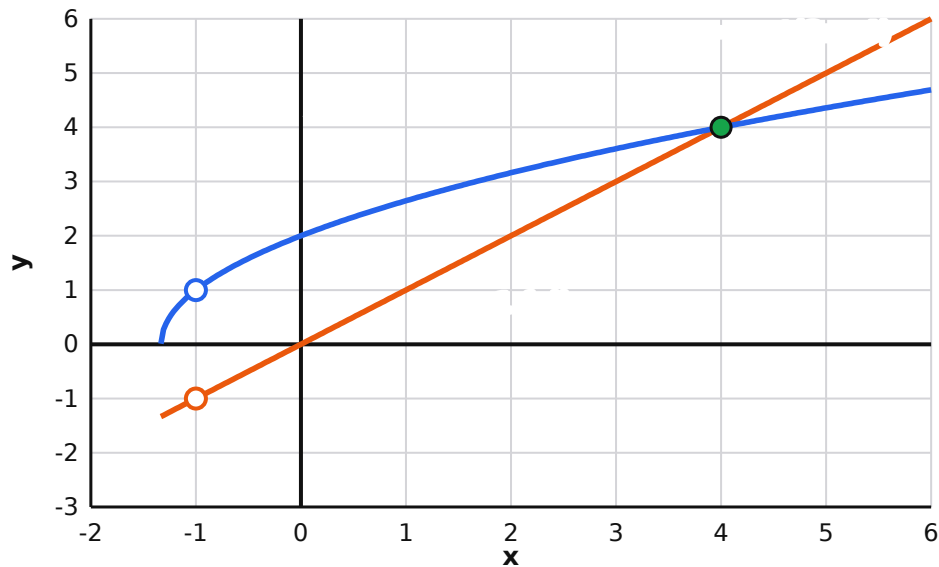
Problem 15: One-step radical equations

- a. Square both sides: $x = 49$.
 b. Cube both sides: $x = -27$.
 c. $x-4=25$, so $x=29$. Check: $\sqrt{25}=5$.
 d. $2x+1=81$, so $x=40$. Check: $\sqrt{81}=9$.

Problem 16: Check for extraneous solutions

Square both sides: $3x+4=x^2$, so $x^2-3x-4=0$ and $(x-4)(x+1)=0$. The candidates are $x=4$ and $x=-1$.

Check $x=4$: $\sqrt{16}=4$, true. Check $x=-1$: $\sqrt{1}=1$, but $x=-1$, false. The solution is $x=4$; $x=-1$ is extraneous.

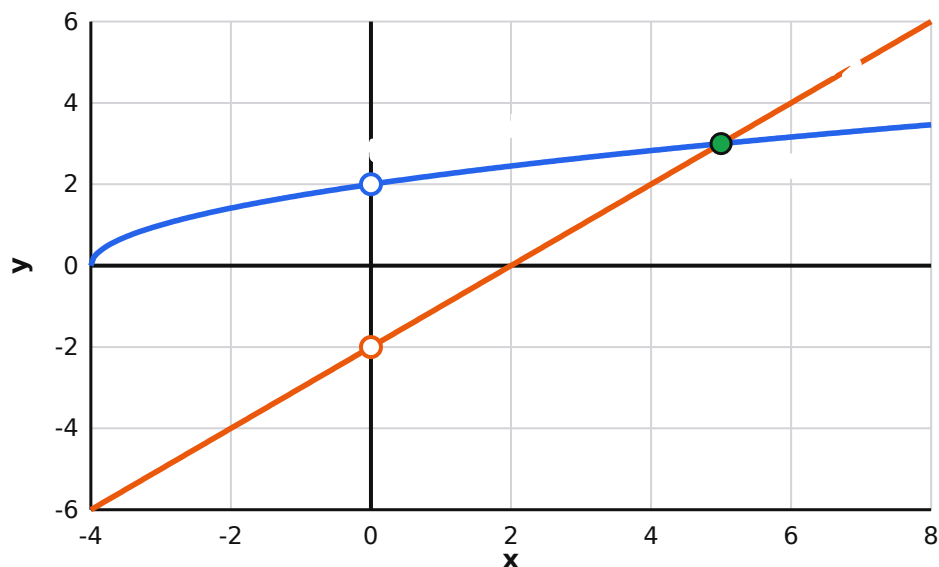


The graphs meet at $(4, 4)$ only; at $x = -1$ the heights are 1 and -1 .

Problem 17: Radical on the right

Square both sides: $x^2 - 4x + 4 = x + 4$, so $x^2 - 5x = 0$ and $x(x - 5) = 0$. The candidates are $x = 0$ and $x = 5$.

Check $x = 0$: $0 - 2 = -2$, but $\sqrt{4} = 2$, false. Check $x = 5$: $5 - 2 = 3$ and $\sqrt{9} = 3$, true. The solution is $x = 5$; $x = 0$ is extraneous.



The line and the curve meet at $(5, 3)$.

Problem 18: Isolate the radical first

a. $\sqrt{x+5} = 7$, so $x+5 = 49$ and $x = 44$.

b. $2\sqrt{x-1} = 8$, $\sqrt{x-1} = 4$, $x-1 = 16$: $x = 17$.

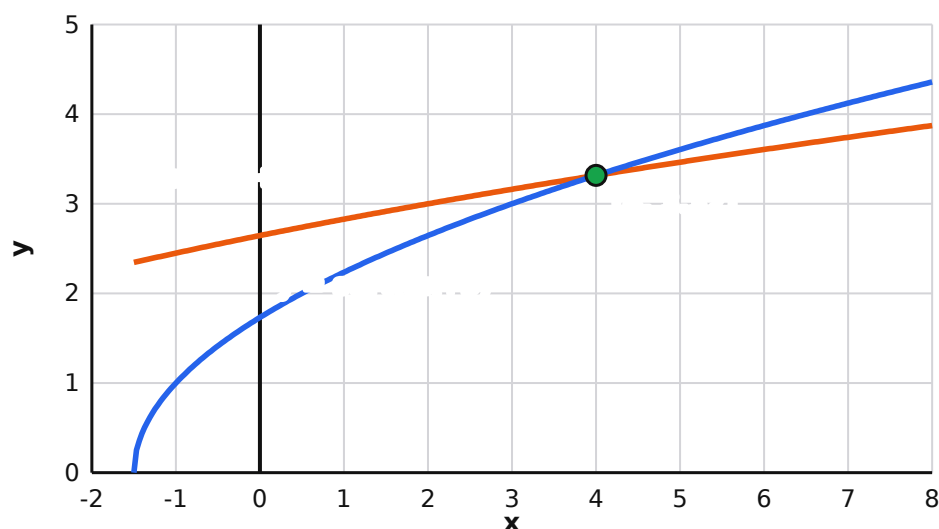
- c. $\sqrt[3]{2x-5}=3$, $2x-5=27$, $2x=32$: $x=16$.
 d. $\sqrt{x+7}=-2$. A principal square root is never negative: **no solution**.

Problem 19: Rational exponent equations

- a. Raise both sides to $\frac{2}{3}$: $x=27^{2/3}=9$. Check: $9^{3/2}=27$. $x=9$
 b. The numerator is even, so there are two solutions: $x=\pm 16^{3/2}=\pm 64$. Check: $64^{2/3}=16$ and $(-64)^{2/3}=(-4)^2=16$. $x=\pm 64$
 c. Raise both sides to the fourth power: $x=81$.
 d. $x-1=\pm 4^{3/2}=\pm 8$, so $x=9$ or $x=-7$. Check: $8^{2/3}=4$ and $(-8)^{2/3}=4$. $x=9$ **or** $x=-7$

Problem 20: Equations with two radicals

- a. $2x+3=x+7$, so $x=4$. Check: both sides equal $\sqrt{11}$.



Both curves reach $\sqrt{11} \approx 3.32$ at $x=4$.

- b. $x+4=3x-10$, so $2x=14$ and $x=7$. Check: both sides equal $\sqrt[3]{11}$.
 c. $5x-1=2x+8$, so $3x=9$ and $x=3$. Check: both sides equal $\sqrt{14}$.

Problem 21: Pendulum period

- a. $T=2\pi\sqrt{\frac{8}{32}}=2\pi\cdot\frac{1}{2}=\pi$. $\pi \approx 3.14$ **seconds**.
 b. $\sqrt{\frac{L}{32}}=1$, so $\frac{L}{32}=1$. $L=32$ **feet**.
 c. $\sqrt{\frac{L}{32}}=2$, so $\frac{L}{32}=4$. $L=128$ **feet**.

Problem 22: Free fall time

- a. $t=\sqrt{\frac{144}{16}}=\sqrt{9}=3$ **seconds**.
 b. $\frac{h}{16}=2.5^2=6.25$, so $h=100$. **100 feet**.
 c. $t=\sqrt{25}=5$ **seconds**.

Problem 23: Skid marks

- a. $s = \sqrt{30(0.75)(40)} = \sqrt{900} = 30$ **mph.**
- b. $45 = \sqrt{30(0.6)d}$, so $2025 = 18d$. $d = 112.5$ **feet.**
- c. $s = \sqrt{720} = \sqrt{144 \cdot 5} = 12\sqrt{5}$. $12\sqrt{5} \approx 26.83$ **mph.**

Problem 24: Distance to the horizon

- a. $d = \sqrt{225} = 15$ **miles.**
- b. $441 = 1.5h$, so $h = 294$ **feet.**
- c. $d = \sqrt{9} = 3$ **miles.**

Problem 25: Cube-shaped box

- a. $e = \sqrt[3]{250} = \sqrt[3]{125 \cdot 2} = 5\sqrt[3]{2}$ **inches.**
- b. $5(1.2599) \approx 6.30$ **inches.**
- c. $e^2 = (5\sqrt[3]{2})^2 = 25\sqrt[3]{4} \approx 25(1.5874) \cdot 25\sqrt[3]{4} \approx 39.69$ **in².**

Problem 26: Bike value

- a. $V = 900(0.64)^{1/2} = 900(0.8) = \$720$.
- b. $0.64^{3/2} = 0.8^3 = 0.512$, so $\$460.80$.
- c. $(0.64)^{1/2} = \frac{576}{900} = 0.64$, so $\frac{t}{2} = 1$. $t = 2$ **years.**
- d. $0.64^{5/2} = 0.8^5 = 0.32768$, so $V = \$294.912$. $\$294.91$