

Algebra 2 · Radicals and Rational Exponents · Lesson

Common Core N-RN.A.1, N-RN.A.2, A-REI.A.2 · mathovore.fr

An n th root of a number a is a number whose n th power equals a , written $\sqrt[n]{a}$ or $a^{1/n}$. A rational exponent joins a root and a power: $a^{m/n} = (\sqrt[n]{a})^m$. With this link you can simplify radicals and solve radical equations, always checking for extraneous solutions.

Algebra 2 Common Core: N-RN.A.1 · N-RN.A.2 · A-REI.A.2

Key facts:

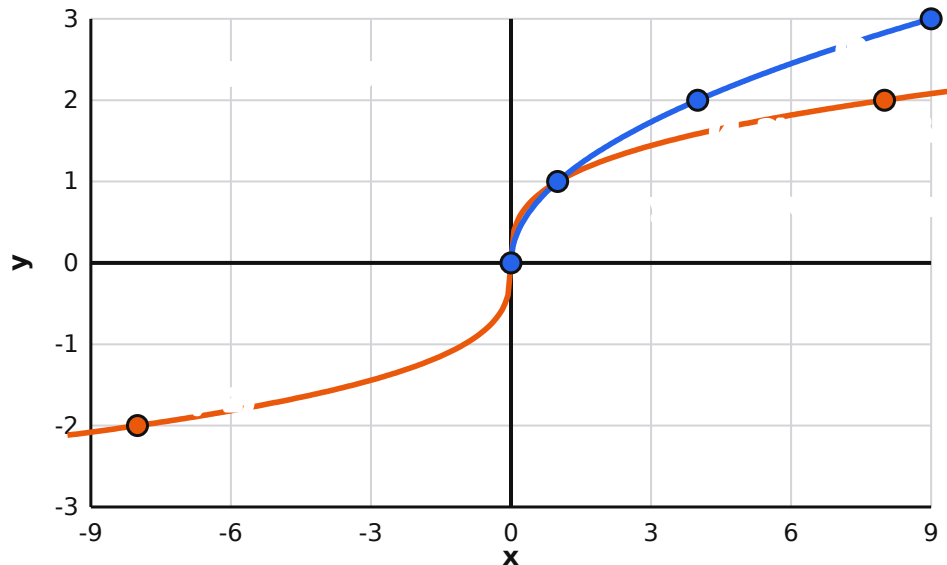
- Standards N-RN.A.1, N-RN.A.2 and A-REI.A.2: rational exponents, radical forms, and radical equations.
- $a^{m/n} = (\sqrt[n]{a})^m$: the denominator is the root index, the numerator is the power.
- An odd root of a negative number is real ($\sqrt[3]{-8} = -2$); an even root of a negative number is not real.
- Simplify by pulling out perfect powers: $\sqrt{72} = 6\sqrt{2}$ and $\sqrt[3]{54} = 3\sqrt[3]{2}$.
- Squaring both sides can create extraneous solutions, so every answer must be checked in the original equation.

What is an n th root?

The **n th root** of a number a is a number that equals a when multiplied by itself n times. It is written $\sqrt[n]{a}$, where n is the **index** and a is the **radicand**. For example, $\sqrt[3]{8} = 2$ because $2^3 = 8$.

Roots of negative numbers

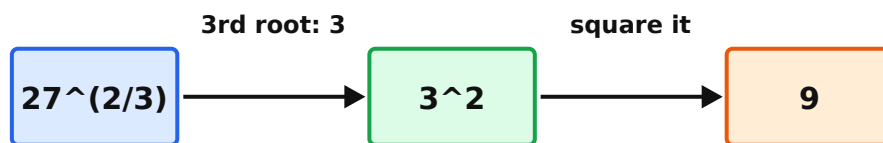
An odd index gives exactly one real root: $\sqrt[3]{-27} = -3$. An even index of a negative radicand has no real root. The radical sign means the **principal** (nonnegative) root, so $\sqrt{16} = 4$, not ± 4 .



The graph of $y = \sqrt[3]{x}$ extends to negative x ; the graph of $y = \sqrt{x}$ starts at the origin.

How do rational exponents relate to radicals?

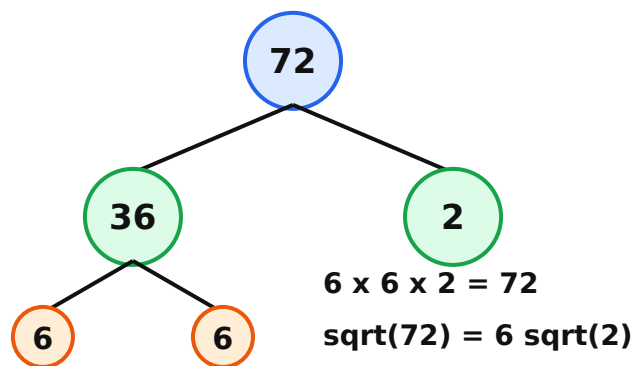
A rational exponent is a radical in disguise: $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$. The denominator is the index and the numerator is the power. A negative exponent means a reciprocal: $a^{-m/n} = \frac{1}{a^{m/n}}$. Take the root first to keep the numbers small, and the usual exponent rules still apply, such as $x^{1/2} \cdot x^{1/3} = x^{5/6}$.



$$\text{Root first, power second: } 27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9.$$

How do you simplify a radical?

Use the product property $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$. Pull out the largest factor that is a perfect n th power. Since $72 = 36 \cdot 2$, we get $\sqrt{72} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$. Likewise $\sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{2} = 3\sqrt[3]{2}$. Like radicals add as like terms do: $2\sqrt{3} + 5\sqrt{3} = 7\sqrt{3}$.



A factor tree shows the perfect square hiding inside 72.

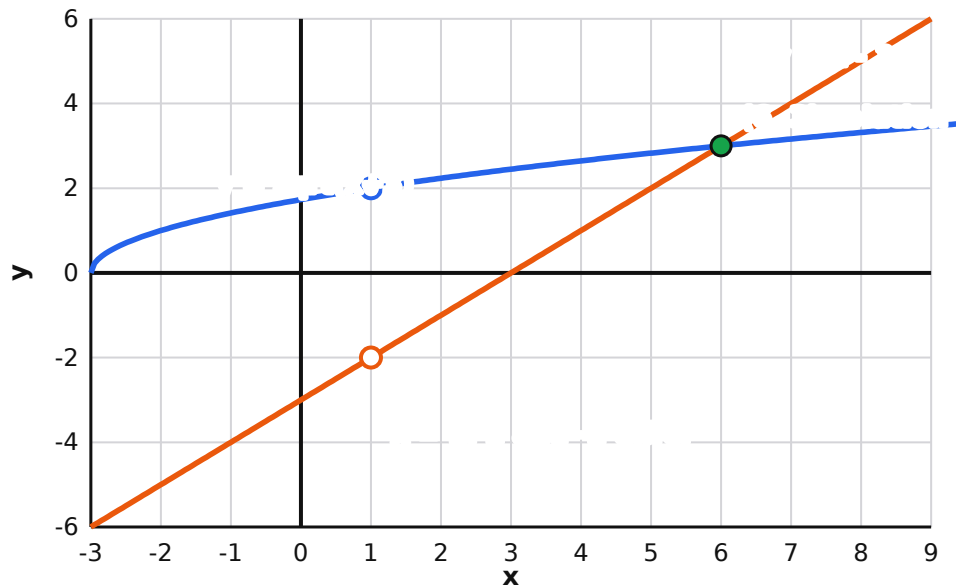
A radical should not stay in a denominator. To **rationalize**, multiply the top and bottom by a radical that makes the denominator a whole number: $\frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$. The table below collects the forms you will convert most often.

Exponent form	Radical form	Value
$49^{1/2}$	$\sqrt{49}$	7
$8^{2/3}$	$(\sqrt[3]{8})^2$	4
$81^{-3/4}$	$\frac{1}{(\sqrt[4]{81})^3}$	$\frac{1}{27}$

How do you solve a radical equation?

1. Isolate the radical on one side.
2. Raise both sides to the power equal to the index.
3. Solve the new equation.
4. Substitute every answer into the *original* equation.

Solve $\sqrt{x+3} = x-3$. Squaring gives $x+3 = x^2-6x+9$, so $x^2-7x+6=0$ and $x=1$ or $x=6$. Check $x=1$: $\sqrt{4}=2$, but $1-3=-2$, so it fails. Check $x=6$: $\sqrt{9}=3$ and $6-3=3$. The only solution is $x=6$; $x=1$ is **extraneous**, an answer created by squaring.



The curve and the line cross only at (6,3).

Radicals and Rational Exponents: frequently asked questions

Is $\sqrt{16}$ equal to 4 or ± 4 ?

The radical sign means the principal root, so $\sqrt{16} = 4$. The equation $x^2 = 16$ has two solutions, $x = \pm 4$, because you add the negative root yourself when solving.

Why can you take the cube root of a negative number but not the square root?

A negative number times itself three times stays negative, so $\sqrt[3]{-27} = -3$. A negative number times itself twice is always positive, so no real number squares to -27 .

How do you evaluate $32^{3/5}$?

Read the denominator as the root and the numerator as the power: $32^{3/5} = (\sqrt[5]{32})^3 = 2^3 = 8$. Taking the root first keeps the numbers small.

What is an extraneous solution?

It is an answer that appears while solving but fails in the original equation. Raising both sides to a power can produce one, for example $x = 1$ in $\sqrt{x+3} = x-3$.

Do the laws of exponents work with fractional exponents?

Yes. Add exponents when multiplying like bases, subtract when dividing, and multiply for a power of a power: $(x^{2/3})^{3/4} = x^{1/2}$. For even roots, assume the variable is nonnegative.