

Algebra 2 · Quadratic Equations with Complex Solutions · Worksheet Answer Key

Common Core N-CN.C.7, A-REI.B.4b · mathovore.fr

Complete worked solutions for the worksheet. Each answer shows the discriminant or the key step, the simplified solutions in bold, and a graph or complex-plane picture when it helps. Check that complex solutions are written in the form $a + bi$.

Algebra 2 Common Core: N-CN.C.7 · A-REI.B.4b

Problem 1: Square root method

Take the square root of both sides: $x = \pm\sqrt{-49} = \pm 7i$.

$$x = \pm 7i$$

Problem 2: Isolate the square

$3x^2 = -27$, so $x^2 = -9$ and $x = \pm 3i$.

$$x = \pm 3i$$

Problem 3: Square of a binomial

$x-2 = \pm 5i$, so $x = 2 \pm 5i$.

$$x = 2 \pm 5i$$

Problem 4: Complete the isolation

$2(x+1)^2 = -18$, $(x+1)^2 = -9$, $x+1 = \pm 3i$.

$$x = -1 \pm 3i$$

Problem 5: Compute discriminants

a) $D = 36 - 20 = 16$. b) $D = 36 - 36 = 0$. c) $D = 36 - 52 = -16$.

a) 16, two real solutions b) 0, one repeated real solution c) -16, two complex solutions

Problem 6: Match graphs to discriminants

A graph that crosses the x-axis twice means two real solutions. One that touches it once means one repeated solution. One that stays above or below it means no real solutions.

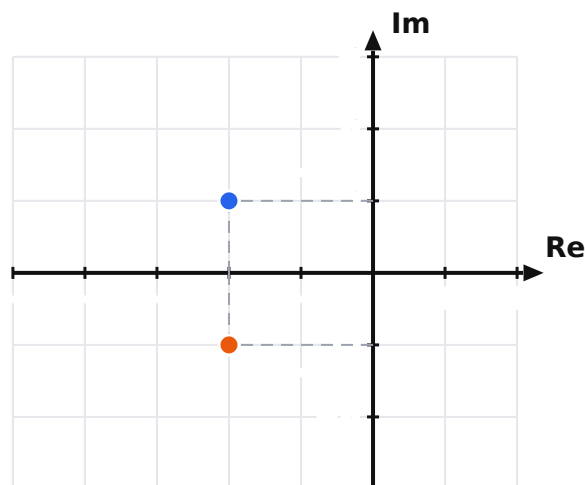
I: $D > 0$, 2 real solutions. II: $D = 0$, 1 real solution. III: $D < 0$, 0 real solutions (two complex).

Problem 7: Graph and solve

The vertex is $(-2, 1)$ and the parabola opens up, so it never reaches the x-axis: no real solutions.

$$D = 16 - 20 = -4, \text{ so } x = \frac{-4 \pm 2i}{2} = -2 \pm i.$$

No real solutions; $x = -2 \pm i$



The solutions $-2 + i$ and $-2 - i$ in the complex plane.

Problem 8: Quadratic formula

$$D = 4 - 40 = -36. \quad x = \frac{2 \pm 6i}{2} = 1 \pm 3i.$$

$$x = 1 \pm 3i$$

Problem 9: Complete the square

$$x^2 + 6x = -13. \text{ Add 9: } (x+3)^2 = -4. \text{ Then } x+3 = \pm 2i.$$

$$x = -3 \pm 2i$$

Problem 10: Larger coefficients

$$D = 64 - 100 = -36. \quad x = \frac{8 \pm 6i}{2} = 4 \pm 3i.$$

$$x = 4 \pm 3i$$

Problem 11: Leading coefficient not 1

$$D = 16 - 40 = -24 \text{ and } \sqrt{-24} = 2i\sqrt{6}. \quad x = \frac{4 \pm 2i\sqrt{6}}{4} = 1 \pm \frac{\sqrt{6}}{2}i.$$

$$x = 1 \pm \frac{\sqrt{6}}{2}i$$

Problem 12: Fractions in the answer

$$D = 4 - 12 = -8 \text{ and } \sqrt{-8} = 2i\sqrt{2}. \quad x = \frac{-2 \pm 2i\sqrt{2}}{6} = \frac{-1 \pm i\sqrt{2}}{3}.$$

$$x = \frac{-1 \pm i\sqrt{2}}{3}$$

Problem 13: Rearrange first

$$\text{Write it as } x^2 - 2x + 17 = 0. \quad D = 4 - 68 = -64. \quad x = \frac{2 \pm 8i}{2} = 1 \pm 4i.$$

$$x = 1 \pm 4i$$

Problem 14: Expand then solve

$$x^2 + 6x + 10 = 0. \quad D = 36 - 40 = -4. \quad x = \frac{-6 \pm 2i}{2} = -3 \pm i.$$

$$x = -3 \pm i$$

Problem 15: Find a constant

$$D = 36 - 4k.$$

$$\text{a) } 36 - 4k = 0 \text{ gives } k = 9. \text{ b) } 36 - 4k < 0 \text{ gives } k > 9.$$

$$\text{a) } k = 9 \text{ b) } k > 9$$

Problem 16: Verify a complex solution

$$(1 - 2i)^2 = 1 - 4i + 4i^2 = -3 - 4i \text{ and } -2(1 - 2i) = -2 + 4i.$$

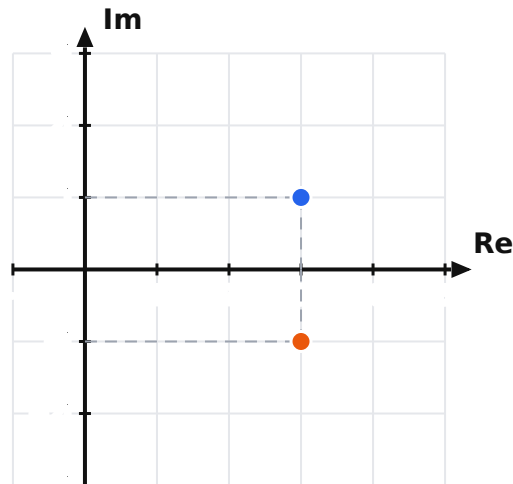
$$(-3 - 4i) + (-2 + 4i) + 5 = 0.$$

The left side is 0, so $1 - 2i$ is a solution.

Problem 17: Build an equation from its roots

The roots are $3+i$ and $3-i$. Their sum is 6 and their product is $9-i^2=10$. For x^2+bx+c , the sum is $-b$ and the product is c .

$b=-6$, $c=10$; the equation is $x^2-6x+10=0$.



The roots $3+i$ and $3-i$.

Problem 18: Conjugate roots

Complex roots of a real quadratic are conjugates, so the other root is $2+5i$. Sum: $4=-b$, so $b=-4$. Product: $4+25=29$, so $c=29$.

Other root $2+5i$; $b=-4$, $c=29$

Problem 19: Does the ball reach 100 feet?

$-16t^2+64t=100$ becomes $4t^2-16t+25=0$ after moving terms and dividing by -4 .

$D=256-400=-144$, so $t=\frac{16\pm12i}{8}=2\pm1.5i$.

The solutions are not real, so the ball never reaches 100 ft. Its maximum height is $h(2)=64$ ft.

Problem 20: Revenue at the school store

$-5p^2+100p=600$ gives $p^2-20p+120=0$. $D=400-480=-80$.

$p=\frac{20\pm4i\sqrt{5}}{2}=10\pm2i\sqrt{5}$.

The solutions are not real, so no price gives \$600. The maximum revenue is \$500 at

$$p = \$10.$$

Problem 21: Find the error

He treated $\sqrt{-4}$ as 2, but $\sqrt{-4} = 2i$. Check $x = 4$: $16 - 24 + 10 = 2 \neq 0$.

Correct: $x = \frac{6 \pm 2i}{2} = 3 \pm i$.

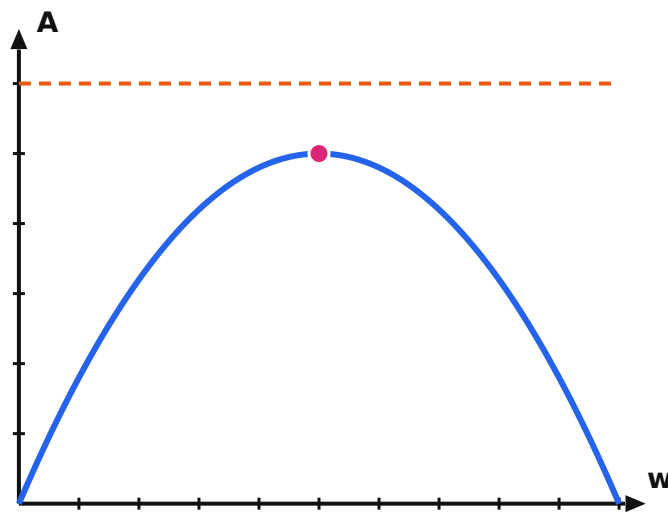
$$x = 3 \pm i$$

Problem 22: Garden with perimeter 20 feet

Length is $10 - w$, so $w(10 - w) = 30$ gives $w^2 - 10w + 30 = 0$.

$$D = 100 - 120 = -20, \text{ so } w = \frac{10 \pm 2i\sqrt{5}}{2} = 5 \pm i\sqrt{5}.$$

The width would not be real, so the garden is impossible. The greatest possible area is $5 \cdot 5 = 25$ sq ft.



The area stays at or below 25 sq ft, so it never reaches 30.

Problem 23: Integer values of k

$D = k^2 - 64$. Non-real solutions need $k^2 - 64 < 0$, so $k^2 < 64$ and $-8 < k < 8$.

The integers from -7 to 7 work, which is 15 values.

$$-7 \leq k \leq 7, \text{ 15 integers}$$

Problem 24: Sum and product of solutions

$$D = 64 - 88 = -24 \text{ and } \sqrt{-24} = 2i\sqrt{6}. \quad x = \frac{-8 \pm 2i\sqrt{6}}{4} = -2 \pm \frac{\sqrt{6}}{2}i.$$

Sum: $-2+(-2)=-4=-\frac{8}{2}$. Product: $(-2)^2-\left(\frac{\sqrt{6}}{2}i\right)^2=4+\frac{6}{4}=\frac{11}{2}$.

$x = -2 \pm \frac{\sqrt{6}}{2}i$; **sum** -4 , **product** $\frac{11}{2}$