

Algebra 2 · Quadratic Equations with Complex Solutions · Lesson

Common Core N-CN.C.7, A-REI.B.4b · mathovore.fr

A quadratic equation with real coefficients has complex solutions when its discriminant $b^2 - 4ac$ is negative. The quadratic formula then gives two conjugate solutions $p \pm qi$, because the square root of a negative number is i times a real square root. The graph never touches the x-axis.

Algebra 2 Common Core: N-CN.C.7 · A-REI.B.4b

Key facts:

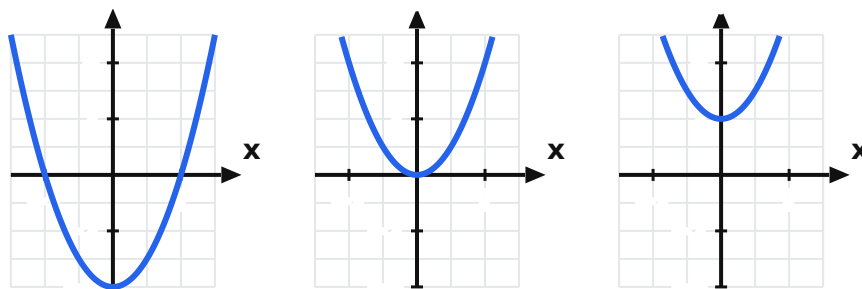
- The discriminant is $D = b^2 - 4ac$.
- $D > 0$: two real solutions; $D = 0$: one repeated real solution; $D < 0$: two complex solutions.
- For $D < 0$, $\sqrt{D} = i\sqrt{-D}$, so the solutions have the form $p \pm qi$.
- Complex solutions of a real quadratic always come as a conjugate pair.
- Square roots, completing the square, and the formula all reach the same solutions.

What does the discriminant tell you?

For $ax^2+bx+c=0$ with real coefficients and $a \neq 0$, the **discriminant** is $D = b^2 - 4ac$. Its sign decides the kind of solutions.

Discriminant	Solutions	Graph
$D > 0$	two different real numbers	crosses the x-axis twice
$D = 0$	one repeated real number	touches the x-axis once
$D < 0$	two complex numbers, conjugates of each other	never meets the x-axis

Computing D first costs one line and tells you what to expect. For $x^2 - 6x + 5 = 0$, $D = 16$ is positive, so the solutions are real. For $x^2 - 6x + 13 = 0$, $D = -16$ is negative, so the solutions are complex. The graph of the related function shows the same fact: it either crosses, touches, or misses the x-axis.

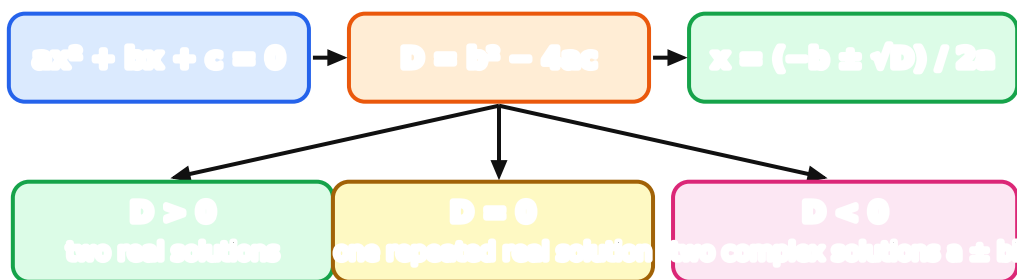


The sign of the discriminant $D = b^2 - 4ac$ decides how many times the parabola meets the x-axis.

How do you use the quadratic formula when D is negative?

Quadratic formula

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. When $D < 0$, write $\sqrt{D} = i\sqrt{-D}$. The two solutions are the **complex conjugates** $p+qi$ and $p-qi$.



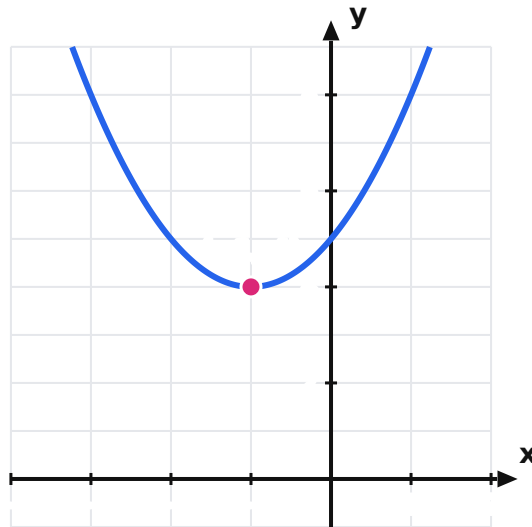
Compute the discriminant first: it tells you what kind of solutions to expect.

Worked example: solve $x^2 - 4x + 13 = 0$.

1. Identify $a=1$, $b=-4$, $c=13$.
2. Compute $D = (-4)^2 - 4(1)(13) = 16 - 52 = -36$. It is negative, so expect complex solutions.
3. Substitute: $x = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2}$.
4. Divide both parts by 2: $x = 2 \pm 3i$.

When can you solve with square roots or completing the square?

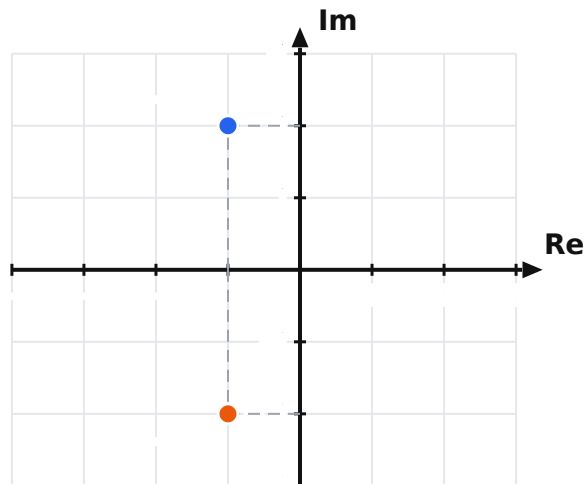
If the equation has the form $x^2 = -k$ with $k > 0$, then $x = \pm i\sqrt{k}$. For $(x+3)^2 = -4$ take the square root of both sides to get $x+3 = \pm 2i$, so $x = -3 \pm 2i$. To solve $x^2 + 6x + 13 = 0$, move the constant, add 9 to both sides to get $(x+3)^2 = -4$, and finish the same way.



The graph of $y = x^2 + 2x + 5$ stays above the x-axis, so $x^2 + 2x + 5 = 0$ has no real solutions.

Why do the solutions come in pairs?

Always check a complex solution by substituting it and using $i^2 = -1$. For $2+3i$ in $x^2-4x+13$, $(2+3i)^2 = -5+12i$, and $(-5+12i)-4(2+3i)+13=0$. The formula gives $p \pm qi$, so complex solutions of a real-coefficient quadratic always appear as a conjugate pair. In the complex plane they are mirror images across the real axis. For $x^2+2x+5=0$, $D=-16$ and $x = -1 \pm 2i$.



The complex solutions $-1 \pm 2i$ are mirror images across the real axis.

Quadratic Equations with Complex Solutions: frequently asked questions

What does a negative discriminant mean?

It means the quadratic has no real solutions and exactly two complex solutions. The parabola does not cross the x-axis. For $x^2+2x+5=0$ the discriminant is -16 .

Why are complex solutions always a conjugate pair?

The quadratic formula has the shape $\frac{-b \pm \sqrt{D}}{2a}$. When $D < 0$ the second part is imaginary, and the plus and minus versions are conjugates.

Can I check a complex solution?

Yes. Substitute it and use $i^2 = -1$. For $2+3i$ in $x^2-4x+13$: $(-5+12i)-8-12i+13=0$.

What if the discriminant is zero?

Then $\sqrt{D}=0$ and the formula gives one solution, $x = -\frac{b}{2a}$. It is repeated, and the vertex of the parabola sits on the x-axis.

Is the discriminant useful before solving?

Yes. It tells you what to expect, so you can check your answer. A negative D means a result such as $3 \pm 4i$, not real numbers.