

Algebra 2 · Probability: Sample Spaces, Independence and Conditional Probability · Worksheet Answer Key

Common Core S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7 · mathovore.fr

Each solution shows the rule used, the substitution and the result in bold. If your answer differs, check whether you used the right denominator for conditional probabilities and whether you subtracted the overlap in the addition rule.

Algebra 2 Common Core: S-CP.A.1 · S-CP.A.2 · S-CP.A.3 · S-CP.A.4 · S-CP.A.5 · S-CP.B.6 · S-CP.B.7

Problem 1: Sample space of two actions

- a. Sample space: $\{H1, HH3, H4, H5, H6, T1, T2, T3, T4, T5, T6\}$. There are **12 outcomes**.
- b. The event is $\{H2, H4, H6\}$, so $P = \frac{3}{12} = \frac{1}{4}$.

Problem 2: Events as subsets

- a. $A = \{2, 4, 6\}$ and $B = \{4, 5, 6\}$.
- b. " A and B " = $\{4, 6\}$; " A or B " = $\{2, 4, 5, 6\}$; "not A " = $\{1, 3, 5\}$.
- c. $P(A \text{ or } B) = \frac{4}{6} = \frac{2}{3}$.

Problem 3: Use the complement

- a. $1 - 0.35 = 0.65$
- b. $1 - \frac{3}{8} = \frac{5}{8}$
- c. The only outcome with no heads is TTT, with probability $\frac{1}{8}$. So $1 - \frac{1}{8} = \frac{7}{8}$.

Problem 4: Cards from a deck

- a. $\frac{13}{52} = \frac{1}{4}$
- b. $\frac{4}{52} = \frac{1}{13}$
- c. There are 3 face cards in each of 2 red suits: $\frac{6}{52} = \frac{3}{26}$
- d. $1 - \frac{13}{52} = \frac{39}{52} = \frac{3}{4}$

Problem 5: Apply the addition rule

- a. $0.5 + 0.3 - 0.1 = 0.7$
- b. $\frac{3}{8} + \frac{4}{8} - \frac{1}{8} = \frac{6}{8} = \frac{3}{4}$

Problem 6: Find the overlap

Rearrange the addition rule: $P(A \text{ and } B) = P(A) + P(B) - P(A \text{ or } B) = 0.5 + 0.6 - 0.8 = 0.3$.

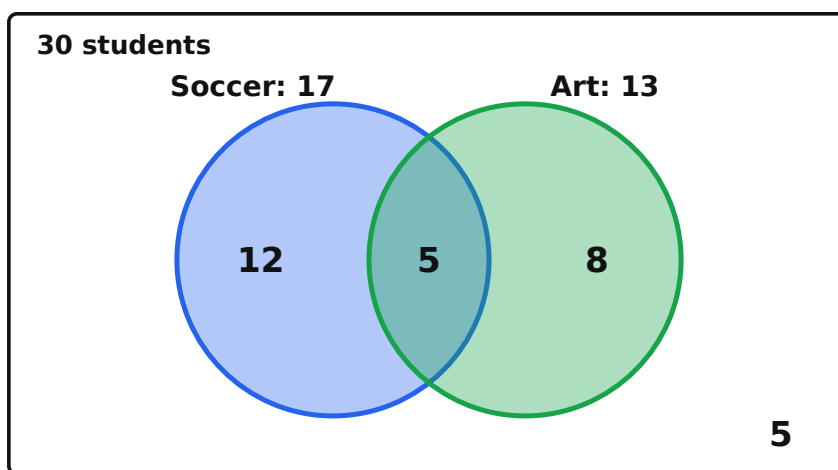
Problem 7: Heart or king

$P(\text{heart}) = \frac{13}{52}$, $P(\text{king}) = \frac{4}{52}$, and the king of hearts is in both: $\frac{1}{52}$.

$$\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13}$$

Problem 8: Read a Venn diagram

Soccer total $12 + 5 = 17$; art total $8 + 5 = 13$.



Totals for each circle.

- a. $\frac{17}{30}; \frac{17}{30}$
- b. $\frac{13}{30}; \frac{13}{30}$
- c. $\frac{5}{30} = \frac{1}{6}$
- d. $\frac{17}{30} + \frac{13}{30} - \frac{5}{30} = \frac{25}{30} = \frac{5}{6}$
- e. $\frac{5}{30} = \frac{1}{6}$

Problem 9: Two dice

- a. The sum is 8 for (2,6),(3,5),(4,4),(5,3),(6,2): $\frac{5}{36} \cdot \frac{5}{36}$
- b. Sums 10, 11, 12 occur $3+2+1=6$ ways: $\frac{6}{36} = \frac{1}{6}$
- c. Six doubles: $\frac{6}{36} = \frac{1}{6}$

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Sum of at least 10 is highlighted

The six outcomes for part b.

Problem 10: Mutually exclusive events

- They cannot happen together: 0
- $0.25 + 0.4 - 0 = 0.65$
- No. Independence needs $P(A \text{ and } B) = 0.25 \cdot 0.4 = 0.1$, but the intersection is 0. **Not independent**

Problem 11: Test independence by the product

- $0.4 \cdot 0.5 = 0.2 = P(A \text{ and } B)$: **independent**
- $0.6 \cdot 0.5 = 0.30$, but $P(A \text{ and } B) = 0.35$: **not independent**

Problem 12: Area model

The regions come from products: $0.6 \cdot 0.5 = 0.30$, $0.6 \cdot 0.5 = 0.30$, $0.4 \cdot 0.5 = 0.20$, $0.4 \cdot 0.5 = 0.20$.

	A: 0.6	not A: 0.4
not B: 0.5	A, not B 0.30	neither 0.20
B: 0.5	A and B 0.30	B, not A 0.20

Each area is a product of two probabilities.

- The violet region: 0.30
- $0.30 + 0.30 + 0.20 = 0.80$ (or $0.6 + 0.5 - 0.3$)
- The gray region: 0.20

Problem 13: Coin and die

- $\frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$
- $\frac{1}{2} \cdot \frac{3}{6} = \frac{1}{4}$
- Numbers 5 and 6: $\frac{1}{2} \cdot \frac{2}{6} = \frac{1}{6}$

Problem 14: Read a two-way table

- $\frac{50}{100} = \frac{1}{2}$
- $\frac{54}{100} = \frac{27}{50}$
- $\frac{30}{100} = \frac{3}{10}$
- $\frac{50 + 54 - 30}{100} = \frac{74}{100} = \frac{37}{50}$

Problem 15: Conditional from a table

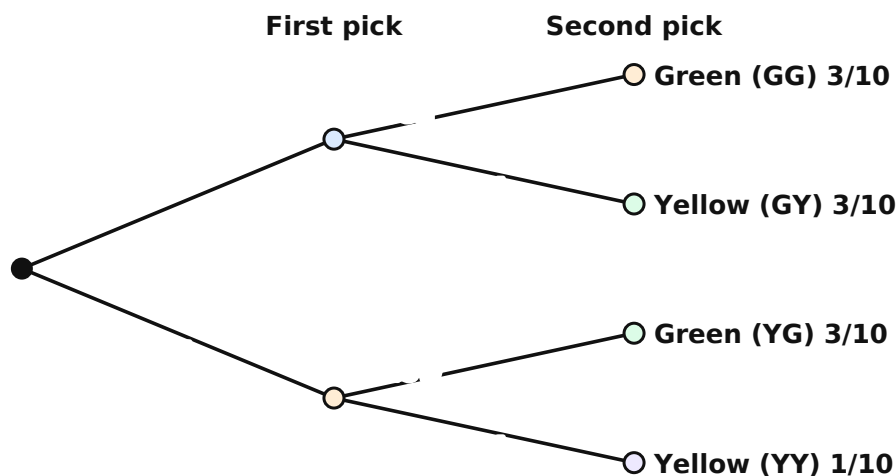
- Use the grade 9 column (50 students): $\frac{30}{50} = \frac{3}{5}$
- Use the pet row (54 students): $\frac{30}{54} = \frac{5}{9}$
- Grade 10 column: $\frac{24}{50} = \frac{12}{25}$

Problem 16: Conditional probability formula

- $\frac{0.12}{0.4} = 0.3$
- $\frac{0.12}{0.5} = 0.24$
- $P(A \text{ and } B) = P(A|B)P(B) = 0.75 \cdot 0.2 = 0.15$

Problem 17: Read a tree diagram

- a. $\frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$
b. Green then yellow $\frac{3}{5} \cdot \frac{2}{4} = \frac{3}{10}$ plus yellow then green $\frac{2}{5} \cdot \frac{3}{4} = \frac{3}{10}$; $\frac{6}{10} = \frac{3}{5}$
c. Read the second branch after green: $\frac{2}{4} = \frac{1}{2}$



The four path products add to 1.

Problem 18: Independence in a table

- a. $\frac{60}{100} = 0.6$
b. $\frac{24}{40} = 0.6$
c. Yes. $P(\text{soccer} | \text{grade 7}) = P(\text{soccer}) = 0.6$. Also $0.6 \cdot 0.4 = 0.24 = P(\text{soccer and grade 7})$.
Independent

Problem 19: Free throws

- a. $0.7 \cdot 0.7 = 0.49$
b. $0.3 \cdot 0.3 = 0.09$
c. Make then miss, or miss then make: $2(0.7)(0.3) = 0.42$ (check: $0.49 + 0.09 + 0.42 = 1$)

Problem 20: Language and art classes

- a. Both: 20; Spanish only 50; art only 25. $\frac{70+45-20}{120} = \frac{95}{120} = \frac{19}{24}$
b. $\frac{120-95}{120} = \frac{25}{120} = \frac{5}{24}$
c. $\frac{20}{70} = \frac{2}{7}$ and $\frac{20}{45} = \frac{4}{9}$
d. $P(\text{Spanish}) \cdot P(\text{art}) = \frac{7}{12} \cdot \frac{3}{8} = \frac{7}{32} \approx 0.219$, but $P(\text{both}) = \frac{1}{6} \approx 0.167$. **Not independent**

Problem 21: Flu shot study

- a. $\frac{12}{200} = 0.06$
b. $\frac{45}{300} = 0.15$

- c. Total who caught the flu: $12+45=57$. $\frac{12}{57} = \frac{4}{19}$
 d. No. The flu rate is 0.06 with a shot and 0.15 without, so the shot changes the probability.
Not independent

Problem 22: Marbles without replacement

- a. $\frac{6}{10} \cdot \frac{5}{9} = \frac{30}{90} = \frac{1}{3}$
 b. $\frac{6}{10} \cdot \frac{4}{9} = \frac{24}{90} = \frac{4}{15}$
 c. Both blue: $\frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90}$. Same color: $\frac{30}{90} + \frac{12}{90} = \frac{42}{90} = \frac{7}{15}$

Problem 23: Weekend weather

- a. $0.4 \cdot 0.3 = 0.12$
 b. $0.4 + 0.3 - 0.12 = 0.58$
 c. $0.6 - 0.7 = 0.42$. Check: $1 - 0.58 = 0.42$.

Problem 24: Defective parts

- a. $P(\text{defective}) = 0.6 \cdot 0.02 + 0.4 \cdot 0.05 = 0.012 + 0.020 = 0.032$
 b. $P(A|\text{defective}) = \frac{0.6 \cdot 0.02}{0.032} = \frac{0.012}{0.032} = \frac{3}{8} = 0.375$