

Algebra 2 · Probability: Sample Spaces, Independence and Conditional Probability · Lesson

Common Core S-CP.A.1, S-CP.A.2, S-CP.A.3, S-CP.A.4, S-CP.A.5, S-CP.B.6, S-CP.B.7 · mathovore.fr

A sample space lists all outcomes of a random process, and an event is a subset of it. Probabilities follow rules: the complement rule, the addition rule $P(A)+P(B)-P(A \text{ and } B)$, and conditional probability $P(A|B)$. Events are independent when $P(A \text{ and } B)=P(A)P(B)$, which means one event does not change the other's probability.

Algebra 2 Common Core: S-CP.A.1 · S-CP.A.2 · S-CP.A.3 · S-CP.A.4 · S-CP.A.5 · S-CP.B.6 · S-CP.B.7

Key facts:

- Standards S-CP.A.1 to S-CP.A.5, S-CP.B.6 and S-CP.B.7: events, independence, conditional probability and the addition rule.
- Complement rule: $P(\text{not } A)=1-P(A)$.
- Addition rule: $P(A \text{ or } B)=P(A)+P(B)-P(A \text{ and } B)$; subtract the overlap once.
- Conditional probability: $P(A|B)=\frac{P(A \text{ and } B)}{P(B)}$, the fraction of B that is also in A .
- Test independence by checking $P(A \text{ and } B)=P(A) \cdot P(B)$ or $P(A|B)=P(A)$.

What are a sample space and an event?

The **sample space** is the set of all possible outcomes of a random process. An **event** is a subset of the sample space. When all outcomes are equally likely,

$P(E)=\frac{\text{outcomes in } E}{\text{outcomes in the sample space}}$. Rolling two dice has 36 outcomes, and 6 of them add to 7, so $P(\text{sum is } 7)=\frac{6}{36}=\frac{1}{6}$.

		Die 2					
		1	2	3	4	5	6
Die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

6 of 36 outcomes have sum 7

Each cell shows the sum of the two dice. The orange cells form the event "sum is 7".

How do unions, intersections and complements work?

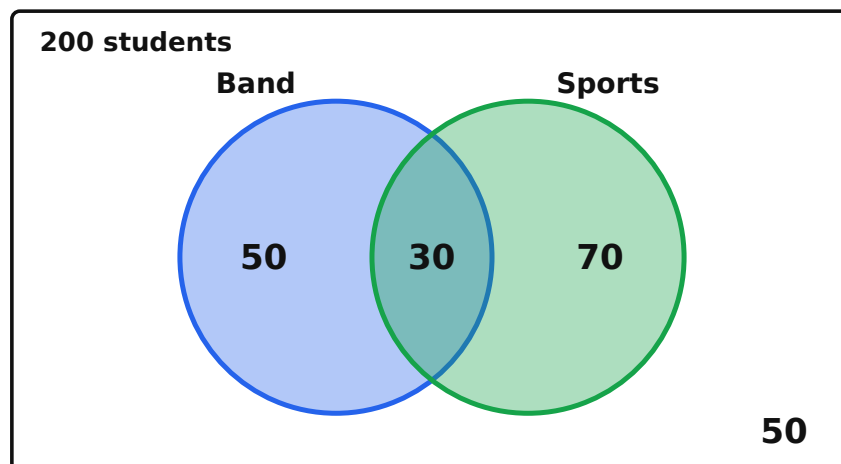
Vocabulary

A and B (intersection): the outcomes that are in both events.

A or B (union): the outcomes in at least one of the events.

Not A (complement): every outcome outside A , with $P(\text{not } A) = 1 - P(A)$.

Suppose 200 students are surveyed. 80 are in band, 100 play a sport and 30 do both. The addition rule avoids counting the overlap twice: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$. Here $0.40 + 0.50 - 0.15 = 0.75$.



The overlap holds the 30 students who do both.

What is conditional probability?

The probability of A given B restricts attention to the outcomes in B : $P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$. A two-way table shows this directly. Among the 80 band students, 30 play a sport, so $P(\text{sports}|\text{band}) = \frac{30}{80} = \frac{3}{8}$.

	Sports	No sports	Total
Band	30	50	80
No band	70	50	120
Total	100	100	200

Read a conditional probability by using only the row or column of the given event.

When are two events independent?

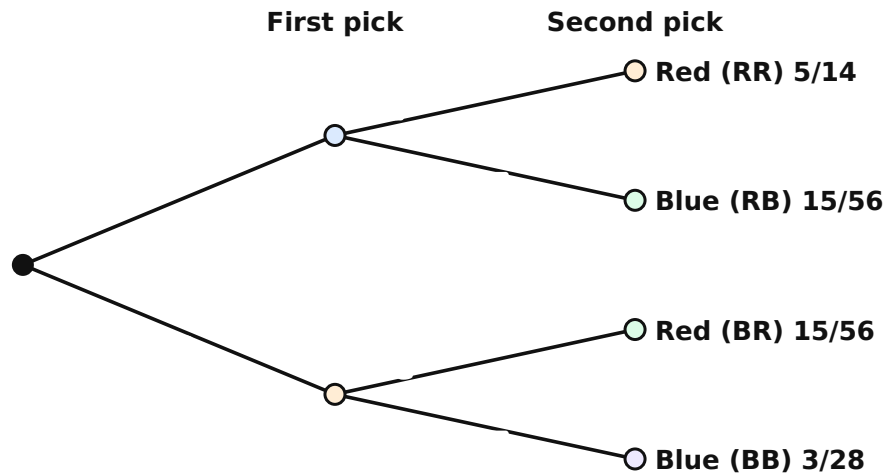
Events are **independent** if knowing one happened does not change the probability of the other. Test with $P(A \text{ and } B) = P(A) \cdot P(B)$, or equivalently $P(A|B) = P(A)$. Band and sports are not independent: $0.40 \cdot 0.50 = 0.20$, but the table gives 0.15.

Rule	Formula
Complement	$P(\text{not } A) = 1 - P(A)$
Addition	$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$
Multiplication	$P(A \text{ and } B) = P(A) \cdot P(B A)$
Independence	$P(A \text{ and } B) = P(A) \cdot P(B)$

How do you use a tree diagram?

1. Draw one branch for each possible first result and write its probability.
2. From each branch, draw the second results with their *conditional* probabilities.
3. Multiply along a path to get the probability of that outcome.
4. Add the paths that belong to the event.

A bag holds 5 red and 3 blue marbles, and two are drawn without replacement. The probability of two reds is $\frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14}$. The probability of two different colors is $\frac{15}{56} + \frac{15}{56} = \frac{15}{28}$.



Multiply along each path; the four outcomes add to 1.

Probability: Sample Spaces, Independence and Conditional Probability: frequently asked questions

What is the difference between independent and mutually exclusive events?

Mutually exclusive events cannot happen together, so $P(A \text{ and } B) = 0$. Independent events do not affect each other, so $P(A \text{ and } B) = P(A)P(B)$. Two events with positive probabilities cannot be both.

Why do you subtract $P(A \text{ and } B)$ in the addition rule?

Outcomes in both events are counted once in $P(A)$ and again in $P(B)$. Subtracting $P(A \text{ and } B)$ removes the double count. If A and B are mutually exclusive, the term is zero.

How do you find a conditional probability from a two-way table?

Use only the row or column for the given event as the denominator. For $P(\text{sports}|\text{band})$, divide the band-and-sports count 30 by the band total 80, which gives $\frac{3}{8}$.

How do you check whether two events are independent?

Compare $P(A \text{ and } B)$ with $P(A) \cdot P(B)$. If the two numbers are equal, the events are independent. Equivalently, check whether $P(A|B)$ equals $P(A)$.

When do the probabilities on a tree diagram change?

They change when the first result affects the second, as when drawing without replacement. Then the second-level branches show conditional probabilities. With replacement, the second-level probabilities repeat the first level.