

Algebra 2 · Polynomial Functions and Their Graphs · Lesson

Common Core F-IF.C.7c, A-APR.B.3, F-IF.B.4 · mathovore.fr

A polynomial function is a sum of terms ax^k with whole-number exponents. Its degree and leading coefficient set the end behavior, and its zeros, read from factored form, show where the graph meets the x-axis. Odd multiplicity crosses, even multiplicity touches, and a degree- n graph has at most $n-1$ turning points.

Algebra 2 Common Core: F-IF.C.7c · A-APR.B.3 · F-IF.B.4

Key facts:

- Standards F-IF.C.7c, A-APR.B.3 and F-IF.B.4: graph polynomials using zeros and end behavior, and interpret key features.
- Even degree: both ends point the same way. Odd degree: the ends point opposite ways. The sign of a_n picks up or down.
- A zero of odd multiplicity is a crossing; a zero of even multiplicity is a touch point.
- A polynomial of degree n has at most n real zeros and at most $n-1$ turning points.
- The y-intercept of f is $f(0)$; in factored form, multiply the constant parts.

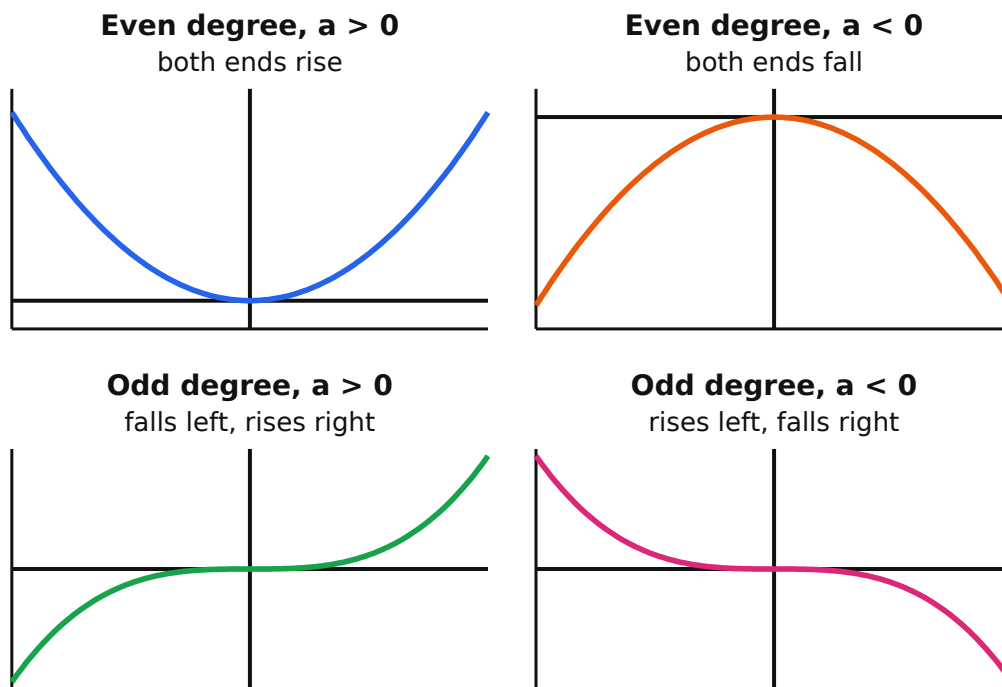
What is a polynomial function?

A **polynomial function** is a sum of terms ax^k whose exponents are whole numbers, such as $f(x) = 4x^3 - 2x^2 + 7$. In general the highest term is $a_n x^n$. The exponent n of the highest term is the **degree**, and its coefficient a_n is the **leading coefficient**. Expressions like $\frac{2}{x}$ or $\sqrt{x}$ are not polynomials.

How do degree and leading coefficient control the ends?

For large $|x|$, only the highest term matters, so the ends of the graph depend on two facts.

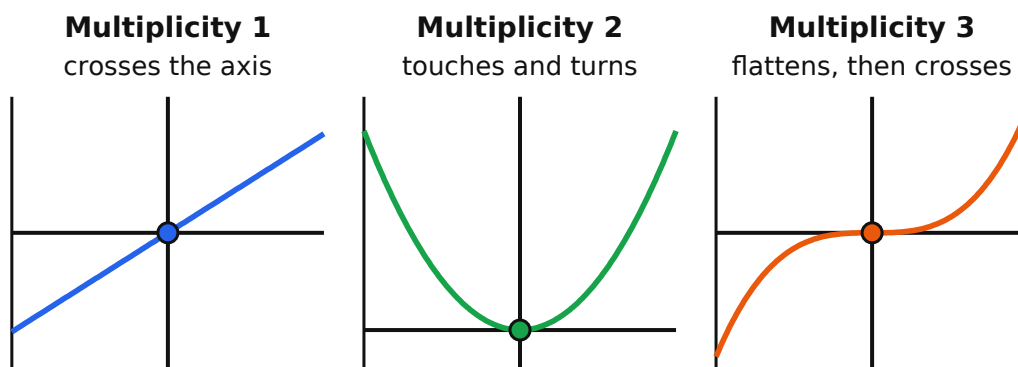
	$a_n > 0$	$a_n < 0$
Even degree	rises left, rises right	falls left, falls right
Odd degree	falls left, rises right	rises left, falls right



End behavior of x^2 , $-x^2$, x^3 and $-x^3$.

What do zeros and multiplicity tell you?

In factored form $f(x) = a(x-r_1)^{m_1}(x-r_2)^{m_2}$, each r is a **zero** and each exponent m is its **multiplicity**. If m is odd the graph crosses the x-axis at r ; if m is even it touches the axis and turns around. The degree equals the sum of the multiplicities.



Multiplicity decides how the graph meets the axis.

How many turning points can a graph have?

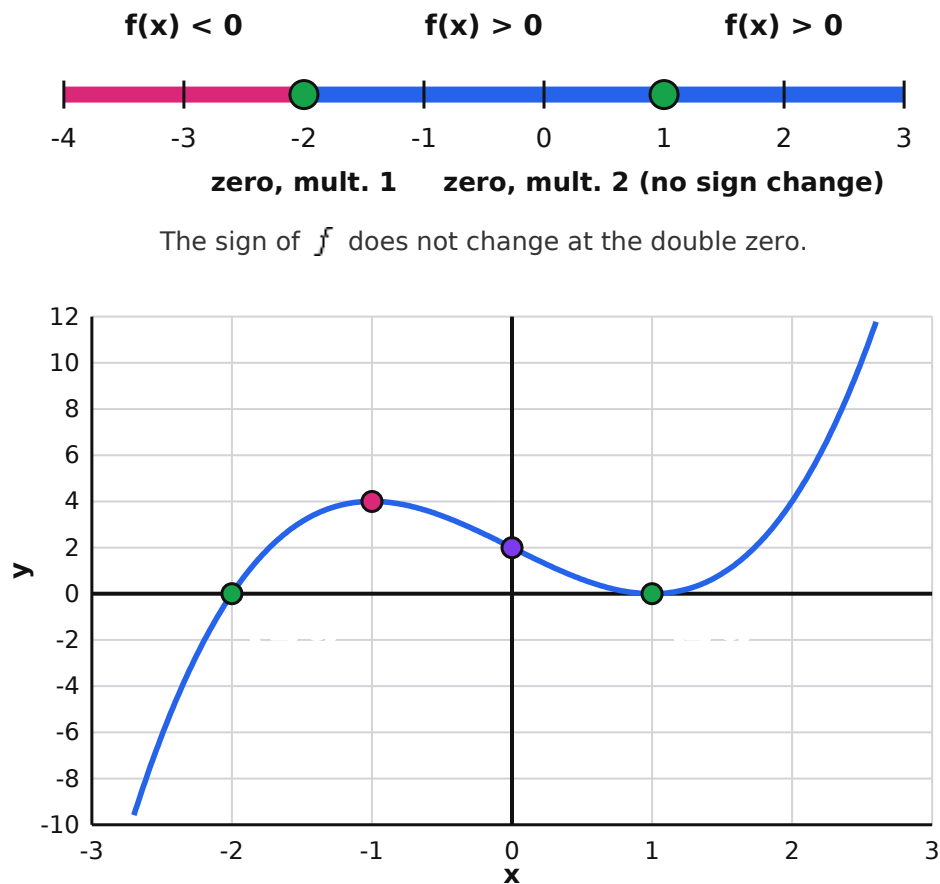
A polynomial of degree n has at most $n-1$ **turning points**, where the graph changes from rising to falling or the reverse. A turning point is a relative maximum or relative minimum.

How do you sketch a graph from factored form?

1. Find the zeros and their multiplicities.

2. Use the degree and leading coefficient for the end behavior.
3. Find the y-intercept $f(0)$.
4. Test one point in each interval to get the signs, then draw a smooth curve.

For $f(x) = (x+2)(x-1)^2$: zeros -2 (crosses) and 1 (touches); degree 3 with positive leading coefficient, so it falls left and rises right; $f(0) = 2 \cdot 1 = 2$. Testing $x = -3$ gives -16 , and $x = 2$ gives 4 .



The graph of $f(x) = (x+2)(x-1)^2$ crosses at -2 and touches at 1 .

Polynomial Functions and Their Graphs: frequently asked questions

How do you find the degree of a polynomial in factored form?

Add the multiplicities of all the factors, counting each linear factor as degree 1. For example, $x(x-2)^2(x+5)^3$ has degree $1+2+3=6$.

What does multiplicity mean for a graph?

It tells how many times a factor repeats. An odd multiplicity makes the graph cross the x-axis, and an even multiplicity makes it touch the axis and turn around. A multiplicity of 3 flattens the graph at the zero.

How many zeros can a polynomial have?

A degree- n polynomial has at most n real zeros, counting multiplicity. An odd-degree polynomial always has at least one real zero because its ends go in opposite directions.

What is a turning point?

It is a point where the graph switches between rising and falling, so it is a relative maximum or minimum. A degree- n polynomial has at most $n-1$ of them.

How do you find the zeros of a polynomial that is not factored?

Factor it first, using a common factor or grouping, then set each factor equal to zero. For example, $x^3-9x = x(x-3)(x+3)$ has zeros 0, 3 and -3 .