

Algebra 2 · Polynomial Division and the Remainder Theorem · Worksheet Answer Key

Common Core A-APR.B.2, A-APR.D.6, A-APR.C.4 · mathovore.fr

This answer key gives a full worked solution for each of the 22 polynomial division problems, including every subtraction in the long division and every row of the synthetic division tables. Check your work against the final bold answer.

Algebra 2 Common Core: A-APR.B.2 · A-APR.D.6 · A-APR.C.4

Problem 1: Divide by a monomial

Divide each term by the monomial.

a) $\frac{12x^3}{4x} = 3x^2$, $\frac{-8x^2}{4x} = -2x$, $\frac{20x}{4x} = 5$.

b) $\frac{9x^4}{3x^2} = 3x^2$, $\frac{6x^3}{3x^2} = 2x$, $\frac{-15x^2}{3x^2} = -5$.

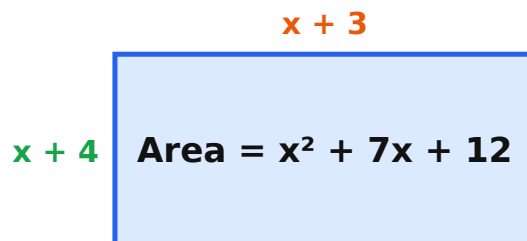
a) $3x^2 - 2x + 5$ b) $3x^2 + 2x - 5$

Problem 2: Missing side of a rectangle

Area equals length times width, so divide the area by $x+3$.

$x^2 \div x = x$. Multiply: $x(x+3) = x^2 + 3x$. Subtract: $4x + 12$. Then $4x \div x = 4$, and $4(x+3) = 4x + 12$, so the remainder is 0.

The other side is $x+4$.



The unknown side is $x+4$.

Problem 3: Long division, zero remainder

a) $x^2 \div x = x$; subtract $x^2 + 2x$ to get $-7x - 14$; $-7x \div x = -7$; subtract $-7x - 14$ to get 0.

b) $x^2 \div x = x$; subtract $x^2 + 4x$ to get $7x + 28$; $7x \div x = 7$; subtract $7x + 28$ to get 0.

a) $x - 7$ **b)** $x + 7$ (both remainders are 0)

Problem 4: Long division with a remainder

a) $x^2 \div x = x$; subtract $x^2 - x$ to get $4x - 2$; $4x \div x = 4$; subtract $4x - 4$ to get 2.

b) $x^2 \div x = x$; subtract $x^2 - 3x$ to get $-3x + 4$; $-3x \div x = -3$; subtract $-3x + 9$ to get -5 .

a) $x + 4 + \frac{2}{x-1}$ **b)** $x - 3 - \frac{5}{x-3}$

Problem 5: Cubic by long division

$x^3 \div x = x^2$; subtract $x^3 - 2x^2$ to get $6x^2 - 3x$.

$6x^2 \div x = 6x$; subtract $6x^2 - 12x$ to get $9x - 18$.

$9x \div x = 9$; subtract $9x - 18$ to get 0.

Quotient $x^2 + 6x + 9$, **remainder** 0.

Problem 6: Division with missing terms

a) Write $x^3 + 0x^2 + 0x - 8$. The quotient terms are x^2 , $2x$, and 4, with products $x^3 - 2x^2$, $2x^2 - 4x$, and $4x - 8$. The remainder is 0.

b) Write $x^3 + 0x^2 + 0x + 27$. The quotient terms are x^2 , $-3x$, and 9, with products $x^3 + 3x^2$, $-3x^2 - 9x$, and $9x + 27$. The remainder is 0.

a) $x^2 + 2x + 4$ **b)** $x^2 - 3x + 9$

Problem 7: Fill in a synthetic division table

Bring down 1. Multiply $1 \cdot 1 = 1$ and add to -6 to get -5 . Multiply $-5 \cdot 1 = -5$ and add to 11 to get 6. Multiply $6 \cdot 1 = 6$ and add to -6 to get 0.

Quotient $x^2 - 5x + 6$, **remainder** 0.

1	1	-6	11	-6
		1	-5	6
	1	-5	6	0
	quotient coefficients			remainder

Completed table: quotient x^2-5x+6 , remainder 0.

Problem 8: Synthetic division with a remainder

Use $c=3$ with coefficients 2, -5, 1, 7. Bring down 2. Then $-5+6=1$, $1+3=4$, and $7+12=19$.

Quotient $2x^2+x+4$, **remainder** 19, so the result is $2x^2+x+4+\frac{19}{x-3}$.

Problem 9: Divide by x plus a number

a) $c=-2$: 1; $5-2=3$; $-2-6=-8$; $-24+16=-8$.

b) $c=-1$ with coefficients 1, 0, -3, 1, 5: 1; $0-1=-1$; $-3+1=-2$; $1+2=3$; $5-3=2$.

a) x^2+3x-8 , **remainder** -8 **b)** x^3-x^2-2x+3 , **remainder** 2

Problem 10: Remainder Theorem

a) $p(3)=27-36+6+5=2$.

b) $p(-1)=-1-4-2+5=-2$.

c) $p(2)=8-16+4+5=1$.

a) 2 **b)** -2 **c)** 1

Problem 11: Use the Factor Theorem

a) $f(2)=8-12-20+24=0$, so $x-2$ is a factor.

b) $f(3)=27-27-30+24=-6$, which is not 0, so $x-3$ is not a factor.

c) $f(-3)=-27-27+30+24=0$, so $x+3$ is a factor.

a) yes **b)** no **c)** yes

Problem 12: Find the missing coefficient

By the Factor Theorem, the value at $x=2$ must be 0: $8+4k-8-12=0$, so $4k-12=0$ and $k=3$.

Check: $x^3+3x^2-4x-12=(x+3)(x^2-4)$, which has the factor $x-2$.

$$k=3$$

Problem 13: Divide by a binomial with leading coefficient 2

a) $6x^2 \div 2x = 3x$; subtract $6x^2+9x$ to get $-8x-12$; $-8x \div 2x = -4$; subtract $-8x-12$ to get 0.

b) $4x^2 \div 2x = 2x$; subtract $4x^2-2x$ to get $-2x+5$; $-2x \div 2x = -1$; subtract $-2x+1$ to get 4.

$$\text{a) } 3x-4 \quad \text{b) } 2x-1+\frac{4}{2x-1}$$

Problem 14: Write the division statement

Use $c=-1$: $3; 2-3=-1; -1+1=0; 4-0=4$. The quotient is $3x^2-x+0$ and the remainder is 4.

Check: $(x+1)(3x^2-x)+4=3x^3-x^2+3x^2-x+4=3x^3+2x^2-x+4$.

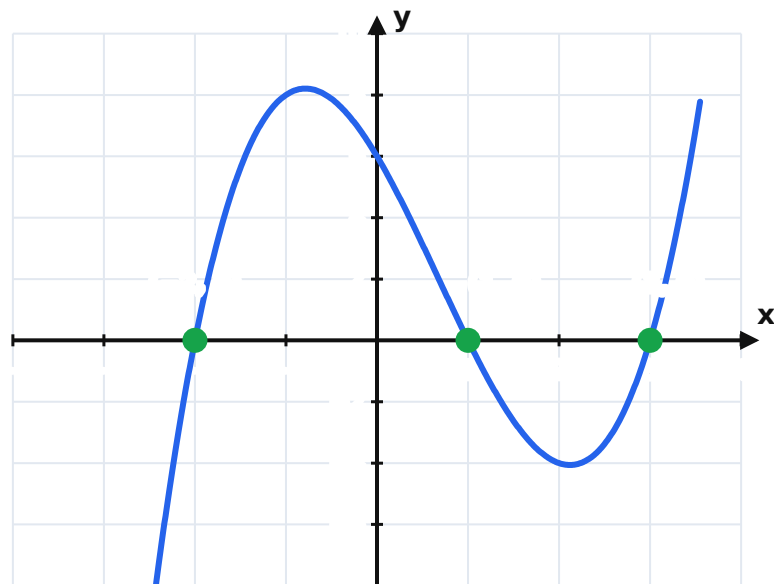
$$p(x)=(x+1)(3x^2-x)+4$$

Problem 15: Factor a cubic from its graph

Divide by $x-1$ with $c=1$: $1; -2+1=-1; -5-1=-6; 6-6=0$. The quotient is x^2-x-6 .

Factor: $x^2-x-6=(x-3)(x+2)$.

$$f(x)=(x-1)(x-3)(x+2); \text{ zeros } -2, 1, 3.$$



All three zeros: $-2, 1$, and 3 .

Problem 16: Find the remaining zeros

Divide by $x+1$ with $c=-1$: $1; -2-1=-3; -7+3=-4; 8+4=12; 12-12=0$. The quotient is $x^3-3x^2-4x+12$.

Divide that by $x-2$ with $c=2$: $1; -3+2=-1; -4-2=-6; 12-12=0$. The quotient is $x^2-x-6=(x-3)(x+2)$.

The other zeros are $x=3$ and $x=-2$. (So $g(x)=(x+1)(x-2)(x-3)(x+2)$.)

Problem 17: Verify a polynomial identity

Left side: $(x^2+y^2)^2=x^4+2x^2y^2+y^4$.

Right side: $(x^2-y^2)^2=x^4-2x^2y^2+y^4$ and $(2xy)^2=4x^2y^2$. Add: $x^4+2x^2y^2+y^4$.

Both sides equal $x^4+2x^2y^2+y^4$, so the identity holds.

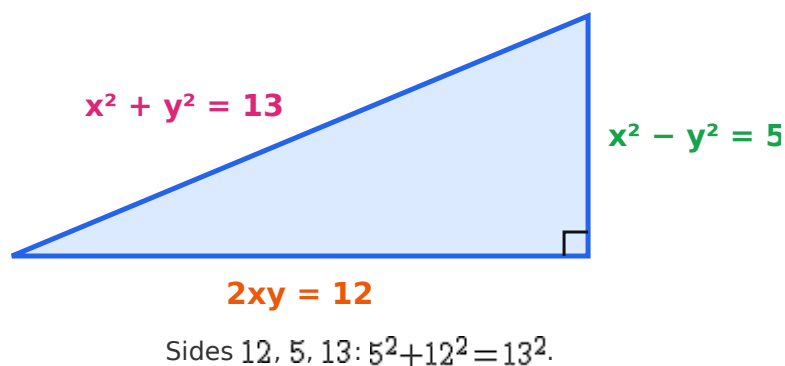
Problem 18: Pythagorean triples from the identity

a) $2xy=2\cdot3\cdot2=12$; $x^2-y^2=9-4=5$; $x^2+y^2=9+4=13$.

b) $5^2+12^2=25+144=169=13^2$.

c) $2xy=20$; $x^2-y^2=25-4=21$; $x^2+y^2=25+4=29$. Check: $21^2+20^2=441+400=841=29^2$.

a) 12, 5, 13 b) $169=13^2$ c) 20, 21, 29



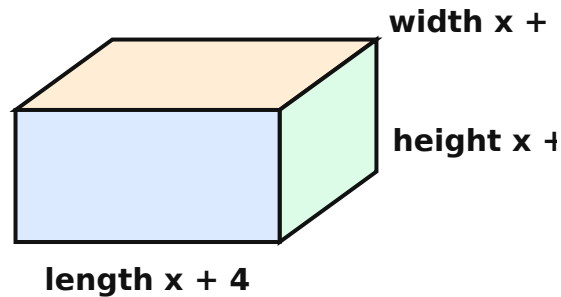
Problem 19: Height of a storage box

a) Height $= V \div (\text{length} \cdot \text{width})$. The product $(x+4)(x+2)=x^2+6x+8$. Divide: $x^3 \div x^2 = x$; subtract x^3+6x^2+8x to get $3x^2+18x+24$; $3x^2 \div x^2 = 3$; subtract $3x^2+18x+24$ to get 0.

b) For $x=3$: length 7 ft, width 5 ft, height 6 ft. Volume $7 \cdot 5 \cdot 6 = 210 \text{ ft}^3$. Check:

$$V(3) = 27 + 81 + 78 + 24 = 210.$$

a) $x+3$ feet b) $7 \times 5 \times 6$ ft, 210 cubic feet



Height $x+3$; for $x=3$ the box is $7 \times 5 \times 6$ feet.

Problem 20: Break-even points

Divide by $x-2$ with $c=2$: $1; -10+2=-8; 31-16=15; -30+30=0$. The quotient is $x^2-8x+15=(x-3)(x-5)$.

The other break-even points are $x=3$ and $x=5$, which means 300 and 500 shirts.

Problem 21: Find two unknown coefficients

Remainder Theorem: $p(1)=1+a+b-6=0$, so $a+b=5$. Also $p(-1)=-1+a-b-6=-10$, so $a-b=-3$.

Add the equations: $2a=2$, so $a=1$. Then $b=4$.

Check: $p(x)=x^3+x^2+4x-6$; $p(1)=1+1+4-6=0$ and $p(-1)=-1+1-4-6=-10$.

$$a=1, b=4$$

Problem 22: Garden dimensions

a) Divide by $2x+1$: $2x^3 \div 2x = x^2$; subtract $2x^3+x^2$ to get $6x^2-5x$; $6x^2 \div 2x = 3x$; subtract $6x^2+3x$ to get $-8x-4$; $-8x \div 2x = -4$; subtract $-8x-4$ to get 0.

$$b) x^2+3x-4=(x+4)(x-1).$$

c) For $x=3$: width 7 ft, length $9+9-4=14$ ft, area $7 \cdot 14 = 98$ ft². Check: $A(3)=54+63-15-4=98$.

a) x^2+3x-4 ft b) $(x+4)(x-1)$ c) 14 ft, 7 ft, 98 square feet