

Algebra 2 · Polynomial Division and the Remainder Theorem · Worksheet

Common Core A-APR.B.2, A-APR.D.6, A-APR.C.4 · mathovore.fr

Name: _____ Date: ____/____/____ Score: _____

These 22 problems practice dividing polynomials with long division and synthetic division, using the Remainder Theorem and the Factor Theorem, and proving polynomial identities (A-APR.B.2, A-APR.D.6, A-APR.C.4).

The problems start with simple quotients, move to cubics and quartics, and end with word problems about boxes, profit, and gardens. Show every step of each division.

Algebra 2 Common Core: A-APR.B.2 · A-APR.D.6 · A-APR.C.4

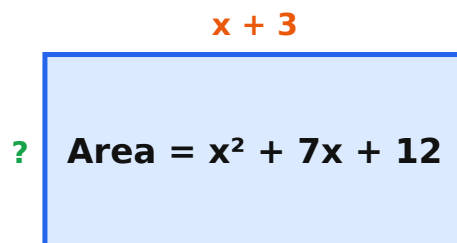
Problem 1: Divide by a monomial

Divide.

- a. $(12x^3 - 8x^2 + 20x) \div 4x$
- b. $(9x^4 + 6x^3 - 15x^2) \div 3x^2$

Problem 2: Missing side of a rectangle

The rectangle has area $x^2 + 7x + 12$ and one side of length $x + 3$. Find the length of the other side.



A rectangle with area $x^2 + 7x + 12$.

Problem 3: Long division, zero remainder

Divide using long division.

- a. $(x^2-5x-14) \div (x+2)$
 b. $(x^2+11x+28) \div (x+4)$

Problem 4: Long division with a remainder

Divide. Write each answer in the form quotient + remainder over divisor.

- a. $(x^2+3x-2) \div (x-1)$
 b. $(x^2-6x+4) \div (x-3)$

Problem 5: Cubic by long division

Divide $x^3+4x^2-3x-18$ by $x-2$ using long division.

Problem 6: Division with missing terms

Rewrite each dividend with zero coefficients for the missing powers, then divide.

- a. $(x^3-8) \div (x-2)$
 b. $(x^3+27) \div (x+3)$

Problem 7: Fill in a synthetic division table

The table sets up synthetic division of $x^3-6x^2+11x-6$ by $x-1$. Complete the table and write the quotient and the remainder.

1	1	-6	11	-6
		?	?	?
	?	?	?	?
	quotient coefficients			remainder

Synthetic division of $x^3-6x^2+11x-6$ by $x-1$; the question marks are to be filled in.

Problem 8: Synthetic division with a remainder

Use synthetic division to divide $2x^3-5x^2+x+7$ by $x-3$.

Problem 9: Divide by x plus a number

Use synthetic division. Remember that $x+2$ means $c=-2$.

- a. $(x^3+5x^2-2x-24) \div (x+2)$

b. $(x^4 - 3x^2 + x + 5) \div (x + 1)$

Problem 10: Remainder Theorem

Let $p(x) = x^3 - 4x^2 + 2x + 5$. Use the Remainder Theorem to find the remainder when $p(x)$ is divided by each divisor. Do not divide.

- a. $x - 3$
- b. $x + 1$
- c. $x - 2$

Problem 11: Use the Factor Theorem

Let $f(x) = x^3 - 3x^2 - 10x + 24$. Decide whether each expression is a factor of $f(x)$. Justify each answer.

- a. $x - 2$
- b. $x - 3$
- c. $x + 3$

Problem 12: Find the missing coefficient

Find the value of k so that $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$.

Problem 13: Divide by a binomial with leading coefficient 2

Divide using long division.

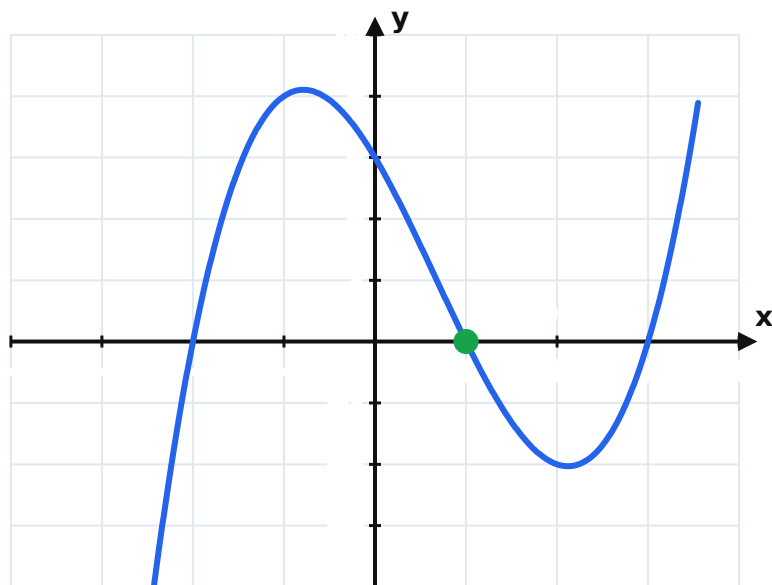
- a. $(6x^2 + x - 12) \div (2x + 3)$
- b. $(4x^2 - 4x + 5) \div (2x - 1)$

Problem 14: Write the division statement

Divide $p(x) = 3x^3 + 2x^2 - x + 4$ by $x + 1$. Then write $p(x)$ in the form $d(x)q(x) + r$.

Problem 15: Factor a cubic from its graph

The graph of $f(x) = x^3 - 2x^2 - 5x + 6$ shows that $x = 1$ is a zero. Use synthetic division to factor $f(x)$ completely and list all zeros.



The graph of $f(x) = x^3 - 2x^2 - 5x + 6$. One zero is marked.

Problem 16: Find the remaining zeros

Given that $x = -1$ and $x = 2$ are zeros of $g(x) = x^4 - 2x^3 - 7x^2 + 8x + 12$, find the other two zeros.

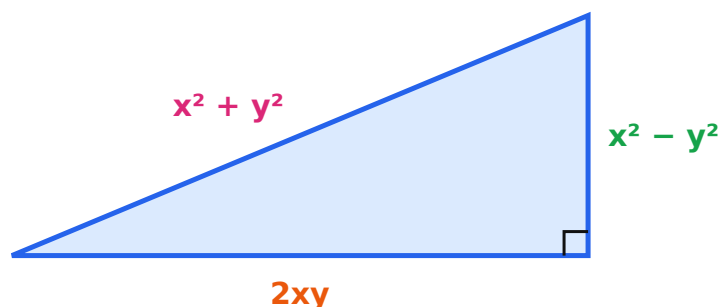
Problem 17: Verify a polynomial identity

Show that $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ for all x and y by expanding both sides.

Problem 18: Pythagorean triples from the identity

The triangle has legs $2xy$ and $x^2 - y^2$ and hypotenuse $x^2 + y^2$. Let $x = 3$ and $y = 2$.

- Find the three side lengths.
- Show that they satisfy the Pythagorean Theorem.
- Find the three side lengths when $x = 5$ and $y = 2$.

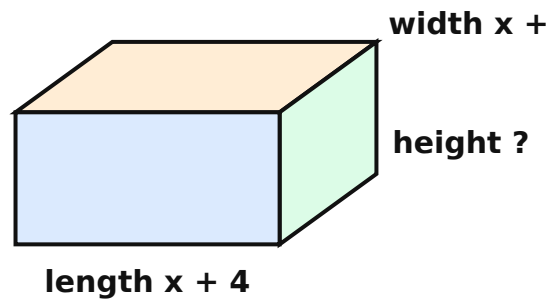


A right triangle built from the identity with $x = 3$ and $y = 2$.

Problem 19: Height of a storage box

The volume of a storage box is $V(x) = x^3 + 9x^2 + 26x + 24$ cubic feet. The length is $x + 4$ feet and the width is $x + 2$ feet.

- Find an expression for the height.
- Find the dimensions and the volume when $x = 3$.



A storage box with volume $x^3 + 9x^2 + 26x + 24$ cubic feet.

Problem 20: Break-even points

Mei models the profit from printing T-shirts for the school store with $P(x) = x^3 - 10x^2 + 31x - 30$, in hundreds of dollars, where x is the number of shirts in hundreds. The store breaks even when $P(x) = 0$, and $x = 2$ is one break-even point. Find the other break-even points.

Problem 21: Find two unknown coefficients

When $p(x) = x^3 + ax^2 + bx - 6$ is divided by $x - 1$, the remainder is 0. When it is divided by $x + 1$, the remainder is -10 . Find a and b .

Problem 22: Garden dimensions

The area of Aisha's rectangular garden is $A(x) = 2x^3 + 7x^2 - 5x - 4$ square feet, and its width is $2x + 1$ feet.

- Find an expression for the length.
- Factor the length completely.
- Find the length, width, and area when $x = 3$.