

Algebra 2 · Polynomial Division and the Remainder Theorem · Test Answer Key

Common Core A-APR.B.2, A-APR.D.6, A-APR.C.4 · mathovore.fr

This answer key shows a full solution and a scoring guide for each of the 9 test questions. Award each point as described, and give partial credit when the method is correct but the arithmetic has a slip.

Algebra 2 Common Core: A-APR.B.2 · A-APR.D.6 · A-APR.C.4

Question 1: Long division (2 points)

$x^3 \div x = x^2$; subtract $x^3 - x^2$ to get $3x^2 - 6x$. $3x^2 \div x = 3x$; subtract $3x^2 - 3x$ to get $-3x + 5$. $-3x \div x = -3$; subtract $-3x + 3$ to get 2.

Quotient $x^2 + 3x - 3$, **remainder** 2.

Scoring: 1 point for the correct quotient, 1 point for the correct remainder.

Question 2: Division with a missing term (2 points)

Write the dividend as $x^4 + 0x^3 - 5x^2 + 3x - 2$ and use $c = -2$: 1 ; $0 - 2 = -2$; $-5 + 4 = -1$; $3 + 2 = 5$; $-2 - 10 = -12$.

Quotient $x^3 - 2x^2 - x + 5$, **remainder** -12 .

-2	1	0	-5	3	-2
		-2	4	2	-10
	1	-2	-1	5	-12

quotient coefficients

remainder

Synthetic division of $x^4 - 5x^2 + 3x - 2$ by $x + 2$.

Scoring: 1 point for including the zero coefficient and using $c = -2$; 1 point for the correct quotient and remainder.

Question 3: Remainder Theorem (2 points)

a) $p(3) = 2(27) - 9 - 21 + 6 = 54 - 9 - 21 + 6 = 30$.

b) $p(-2) = -16 - 4 + 14 + 6 = 0$, so by the Factor Theorem $x+2$ is a factor of $p(x)$ and -2 is a zero.

a) 30 b) $x+2$ is a factor

Scoring: 1 point for each part.

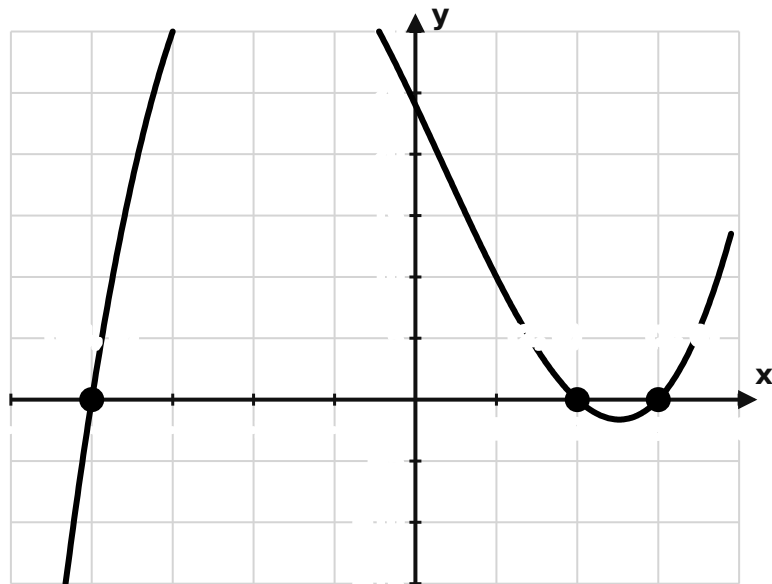
Question 4: Factor Theorem and zeros (3 points)

a) $f(2) = 8 - 4 - 28 + 24 = 0$, so $x-2$ is a factor.

b) Synthetic division with $c=2$: $1; -1+2=1; -14+2=-12; 24-24=0$. The quotient is $x^2+x-12=(x+4)(x-3)$.

c) The zeros are $-4, 2$, and 3 .

a) $f(2)=0$ b) $f(x)=(x-2)(x+4)(x-3)$ c) $-4, 2, 3$



The x-intercepts are $-4, 2$, and 3 .

Scoring: 1 point per part. In part b, 1 point requires all three factors correct.

Question 5: Find k and factor (2 points)

a) $p(-3) = -27 + 9k + 15 + 3 = 9k - 9 = 0$, so $k=1$.

b) With $k=1$, $p(x) = x^3 + x^2 - 5x + 3$. Divide by $x+3$ with $c=-3$: $1; 1-3=-2; -5+6=1; 3-3=0$. The quotient is $x^2-2x+1=(x-1)^2$.

a) $k=1$ b) $(x+3)(x-1)^2$

Scoring: 1 point for each part.

Question 6: Function from a graph (2 points)

a) The zeros are -1 , 2 , and 4 , so $f(x) = (x+1)(x-2)(x-4)$. Check the y-intercept:
 $f(0) = (1)(-2)(-4) = 8$.

b) $f(5) = (6)(3)(1) = 18$.

a) $f(x) = (x+1)(x-2)(x-4)$ b) 18

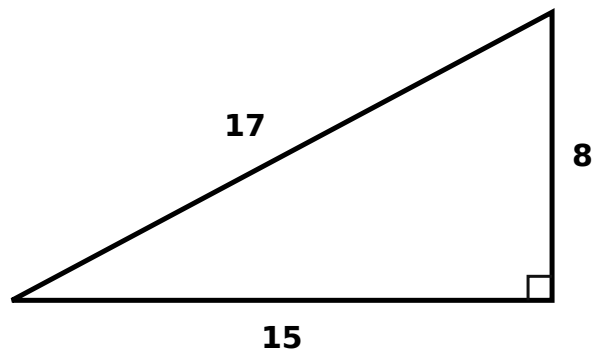
Scoring: 1 point for each part.

Question 7: Polynomial identity (2 points)

a) $(x^2+y^2)^2 = x^4+2x^2y^2+y^4$. Also $(x^2-y^2)^2+(2xy)^2 = x^4-2x^2y^2+y^4+4x^2y^2 = x^4+2x^2y^2+y^4$.
The sides are equal.

b) $x^2-y^2=15$, $2xy=8$, $x^2+y^2=17$. Check: $225+64=289=17^2$.

b) legs 15 and 8, hypotenuse 17



Hypotenuse 17: $8^2+15^2=17^2$.

Scoring: 1 point for a correct expansion of both sides; 1 point for the three side lengths.

Question 8: Dimensions of a box (3 points)

a) The base area is $(x+5)(x+2) = x^2+7x+10$. Divide: $x^3 \div x^2 = x$; subtract x^3+7x^2+10x to get $3x^2+21x+30$; $3x^2 \div x^2 = 3$; subtract $3x^2+21x+30$ to get 0. So $h = x+3$.

b) For $x=4$: $9 \cdot 6 \cdot 7 = 378$ cubic inches. Check: $V(4) = 64+160+124+30 = 378$.

a) $h = x + 3$ inches b) 378 cubic inches

Scoring: part a, 1 point for the base area and 1 point for the division and answer; part b, 1 point.

Question 9: Divide by $2x$ plus 1 (2 points)

$6x^3 \div 2x = 3x^2$; subtract $6x^3 + 3x^2$ to get $-4x^2 + 6x$. $-4x^2 \div 2x = -2x$; subtract $-4x^2 - 2x$ to get $8x + 7$. $8x \div 2x = 4$; subtract $8x + 4$ to get 3.

Quotient $3x^2 - 2x + 4$, remainder 3.

Scoring: 1 point for the correct quotient, 1 point for the correct remainder.

Total
20 points