

Algebra 2 · Polynomial Division and the Remainder Theorem · Lesson

Common Core A-APR.B.2, A-APR.D.6, A-APR.C.4 · mathovore.fr

This algebra 2 polynomial division lesson explains polynomial division and the remainder theorem in one printable page, with definitions, a method and a worked example.

Polynomial division rewrites $p(x)$ as $d(x)q(x)+r(x)$, where the remainder has a smaller degree than the divisor. Long division works for any divisor, and synthetic division is a shortcut for $x-c$. The Remainder Theorem says $p(c)$ is the remainder, and the Factor Theorem says $x-c$ is a factor exactly when $p(c)=0$.

Algebra 2 Common Core: A-APR.B.2 · A-APR.D.6 · A-APR.C.4

Algebra 2 Polynomial Division Lesson: key facts

- Standards A-APR.B.2, A-APR.D.6, and A-APR.C.4: remainders, division of polynomials, and polynomial identities.
- Division statement: $p(x)=d(x)q(x)+r(x)$ where the remainder has smaller degree than the divisor.
- Remainder Theorem: dividing $p(x)$ by $x-c$ leaves remainder $p(c)$.
- Factor Theorem: $x-c$ is a factor of $p(x)$ if and only if $p(c)=0$.
- Identity: $(x^2+y^2)^2=(x^2-y^2)^2+(2xy)^2$ generates Pythagorean triples.

What is polynomial division?

Dividing a polynomial $p(x)$ by a polynomial $d(x)$ works like dividing whole numbers: you find a quotient and a remainder. The result is written as a **division statement**.

Division statement

$p(x)=d(x)q(x)+r(x)$, where the degree of the remainder $r(x)$ is less than the degree of the divisor $d(x)$.

How does long division work?

1. Write both polynomials in descending order, using 0 for any missing power.

2. Divide the leading term of the dividend by the leading term of the divisor and write the result in the quotient.
3. Multiply the whole divisor by that quotient term and subtract the product.
4. Repeat with the new line until its degree is less than the degree of the divisor.

In the example below, $2x^3 \div x = 2x^2$, then $x^2 \div x = x$, then $6x \div x = 6$. The final line, 7, is the remainder.

$$\begin{array}{r}
 \text{2x}^2 \quad \text{x} \quad \text{6} \\
 \text{x} - 2 \overline{) 2x^3 - 3x^2 + 4x - 5} \\
 \underline{-2x^3 \quad 4x^2} \\
 x^2 \quad 4x \\
 \underline{-x^2 \quad 2x} \\
 6x \quad -5 \\
 \underline{-6x \quad 12} \\
 7
 \end{array}$$

Long division of $2x^3 - 3x^2 + 4x - 5$ by $x - 2$: quotient $2x^2 + x + 6$, remainder 7.

What is synthetic division?

Synthetic division is a shortcut for dividing by $x - c$. Write c at the left and the coefficients of $p(x)$ across the top. Bring down the first coefficient, multiply by c , add to the next coefficient, and repeat. For $x - 2$ we use $c = 2$; for $x + 3$ we use $c = -3$.

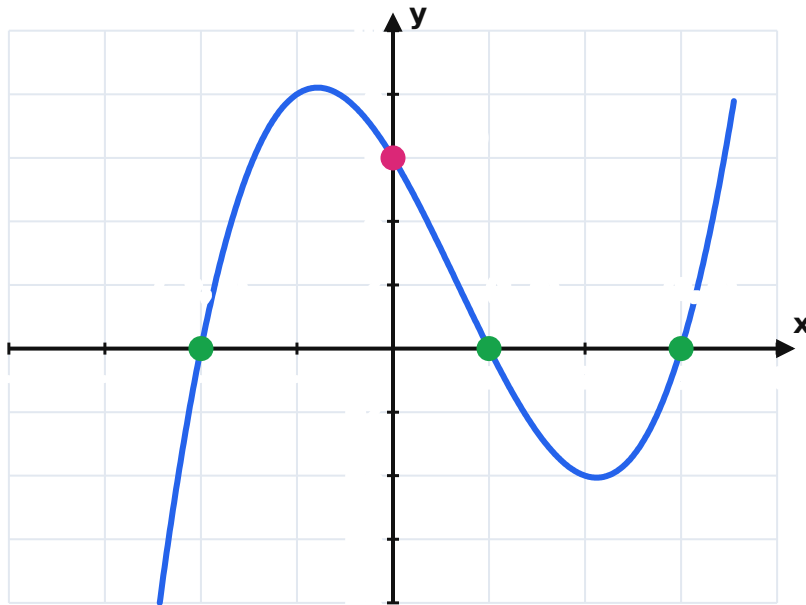
2	2	-3	4	-5
		4	2	12
	2	1	6	7
	quotient coefficients			remainder

Synthetic division with $c = 2$: the bottom row 2, 1, 6 gives the quotient and 7 is the remainder.

What are the Remainder Theorem and the Factor Theorem?

Theorem	Statement
Remainder Theorem	The remainder when $p(x)$ is divided by $x - c$ is $p(c)$.
Factor Theorem	$x - c$ is a factor of $p(x)$ exactly when $p(c) = 0$.

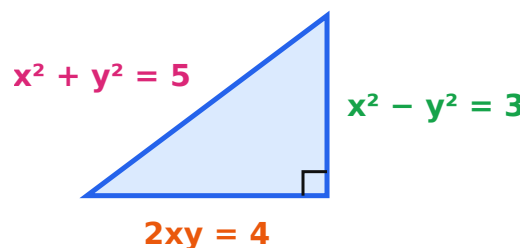
In the example, $p(2) = 2(8) - 3(4) + 4(2) - 5 = 7$, which matches the remainder. For $f(x) = x^3 - 2x^2 - 5x + 6$ we find $f(1) = 0$, so $x - 1$ is a factor. Dividing gives $x^2 - x - 6 = (x - 3)(x + 2)$, so $f(x) = (x - 1)(x - 3)(x + 2)$, and the graph crosses the x-axis at -2 , 1 , and 3 .



Graph of $f(x) = x^3 - 2x^2 - 5x + 6$. Each x-intercept c matches a factor $x - c$.

What is a polynomial identity?

An identity is true for every value of the variables. The identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be proved by expanding both sides: each equals $x^4 + 2x^2y^2 + y^4$. It builds Pythagorean triples: $x = 2$, $y = 1$ gives 5, 3, and 4.



With $x = 2$ and $y = 1$ the identity gives $5^2 = 3^2 + 4^2$.

Polynomial Division and the Remainder Theorem: frequently asked questions

When can you use synthetic division?

Use it when the divisor has the form $x - c$, a linear factor with leading coefficient 1. For a

divisor such as $x+4$, use $c=-4$. For other divisors, use long division.

What do you do when a power is missing?

Insert a zero coefficient for each missing power. To divide x^3-8 , write it as x^3+0x^2+0x-8 . Skipping the zeros shifts every column and gives a wrong quotient.

What does a remainder of zero mean?

A remainder of zero means the divisor divides the polynomial evenly. By the Factor Theorem, $x-c$ is then a factor and c is a zero of the polynomial.

How do you find the remainder without dividing?

Use the Remainder Theorem. Substitute c into the polynomial for a divisor $x-c$. For $p(x)=x^2+3x-2$ and $x-1$, the remainder is $p(1)=2$.

How do you prove a polynomial identity?

Expand or simplify one side until it matches the other side, using only algebra that holds for all values. Checking one number is not a proof, but it is a quick way to catch errors.

How to use this algebra 2 polynomial division lesson

Use this algebra 2 polynomial division lesson before the worksheet, then check what you learned with the quiz.

