

Algebra 2 · Logarithms and Logarithmic Functions · Lesson

Common Core F-LE.A.4, F-BF.B.4a, F-IF.C.7e · mathovore.fr

A logarithm is an exponent. The statement log base b of x equals y means b raised to the power y equals x, so log base 2 of 8 is 3. The logarithmic function is the inverse of the exponential function, and three properties turn products, quotients and powers into sums, differences and multiples.

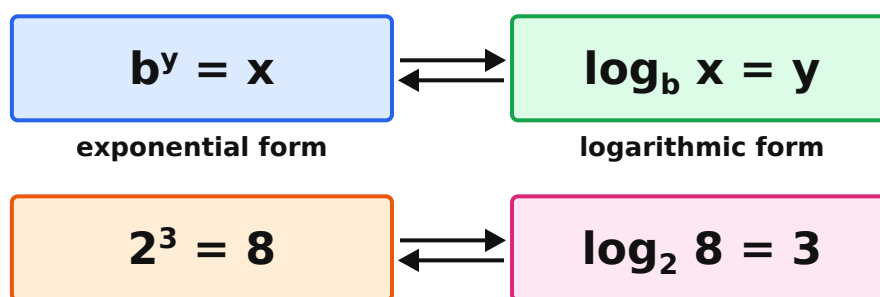
Algebra 2 Common Core: F-LE.A.4 · F-BF.B.4a · F-IF.C.7e

Key facts:

- Standards F-LE.A.4, F-BF.B.4a and F-IF.C.7e: define logarithms, find inverses, and graph logarithmic functions.
- $\log_b x = y$ is equivalent to $b^y = x$; $\log x$ uses base 10 and $\ln x$ uses base e .
- The graph of $y = \log_b x$ has domain $x > 0$, x-intercept (1,0) and the y-axis as its vertical asymptote.
- Product, quotient and power rules: $\log_b MN = \log_b M + \log_b N$, $\log_b \frac{M}{N} = \log_b M - \log_b N$, $\log_b M^p = p \log_b M$.
- Change of base: $\log_b x = \frac{\ln x}{\ln b}$.

What is a logarithm?

A **logarithm** answers the question "what exponent do I need?" The statement $\log_b x = y$ means exactly the same thing as $b^y = x$, where $b > 0$, $b \neq 1$ and $x > 0$. Since $2^3 = 8$, we get $\log_2 8 = 3$.



The base stays the base. The exponent becomes the logarithm.

A logarithm is an exponent: both forms say the same fact.

Vocabulary

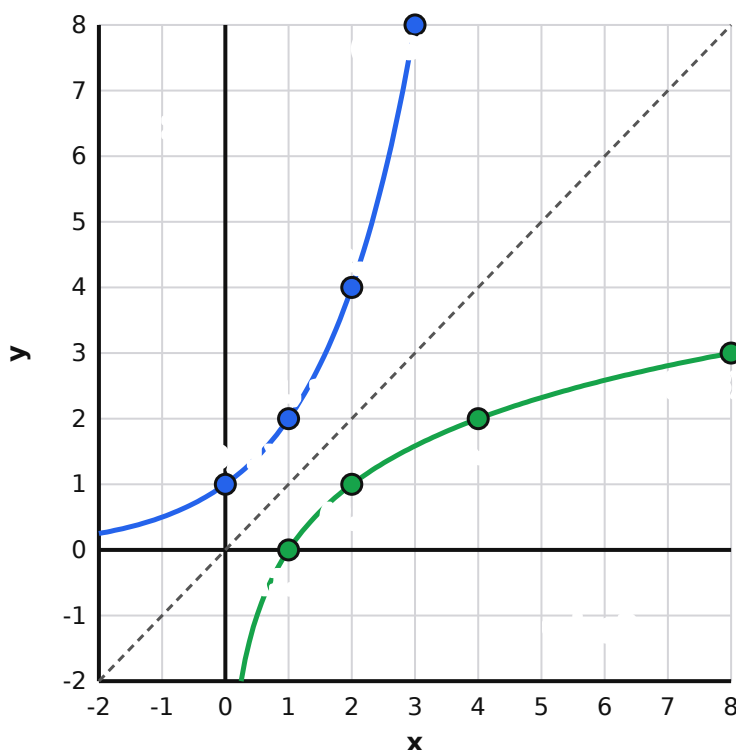
Common logarithm: base 10, written $\log x$. For example $\log 1000 = 3$.

Natural logarithm: base $e \approx 2.718$, written $\ln x$. For example $\ln e^5 = 5$.

Inverse functions: $f(x) = b^x$ and $g(x) = \log_b x$ undo each other, so $\log_b b^x = x$ and $b^{\log_b x} = x$.

What does the graph of a logarithmic function look like?

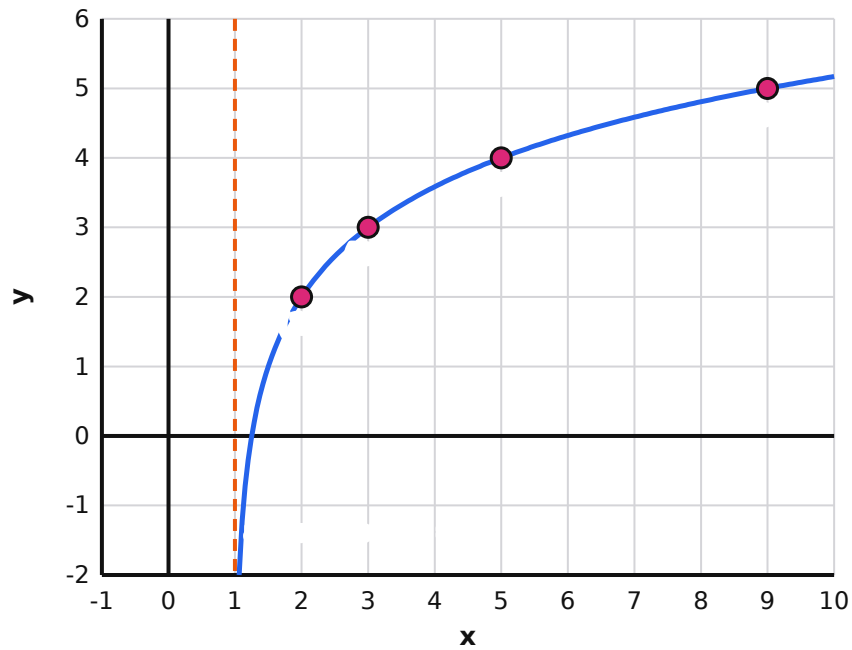
Because $y = \log_b x$ is the inverse of $y = b^x$, its graph is the reflection of the exponential graph across the line $y = x$. Each point (a, c) on one graph matches the point (c, a) on the other.



The graphs of $y = 2^x$ and $y = \log_2 x$ mirror each other across $y = x$.

- Domain: $x > 0$. Range: all real numbers.
- The x-intercept is $(1, 0)$ because $\log_b 1 = 0$.
- The y-axis is a vertical asymptote. The graph rises when $b > 1$.

Transformations work as usual. The graph of $y = \log_2(x-1) + 2$ is the basic graph moved 1 unit right and 2 units up, so its asymptote is $x = 1$ and its domain is $x > 1$.



Graph of $y = \log_2(x - 1) + 2$ with asymptote $x = 1$.

What are the properties of logarithms?

Each property is an exponent rule in disguise. For $M > 0$ and $N > 0$:

Three rules

Product: $\log_b(MN) = \log_b M + \log_b N$

Quotient: $\log_b \frac{M}{N} = \log_b M - \log_b N$

Power: $\log_b M^p = p \log_b M$

Product

$$\log_b(MN) = \log_b M + \log_b N$$

Example

$$\begin{aligned} \log_2 32 \\ = \log_2 8 + \log_2 4 \\ = 3 + 2 = 5 \end{aligned}$$

Quotient

$$\log_b(M/N) = \log_b M - \log_b N$$

Example

$$\begin{aligned} \log_3 (81/9) \\ = \log_3 81 - \log_3 9 \\ = 4 - 2 = 2 \end{aligned}$$

Power

$$\log_b M^p = p \cdot \log_b M$$

Example

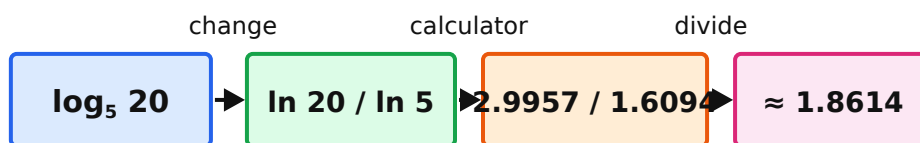
$$\begin{aligned} \log_2 (4^3) \\ = 3 \cdot \log_2 4 \\ = 3 \cdot 2 = 6 \end{aligned}$$

Each rule checked with whole numbers.

How do you change the base of a logarithm?

Calculators have only $\log$ and $\ln$ keys, so use $\log_b x = \frac{\ln x}{\ln b} = \frac{\log x}{\log b}$. Worked example: evaluate $\log_5 20$.

1. Write the quotient $\frac{\ln 20}{\ln 5}$.
2. Find $\ln 20 \approx 2.9957$ and $\ln 5 \approx 1.6094$.
3. Divide: $\frac{2.9957}{1.6094} \approx 1.8614$.
4. Check: $5^{1.8614} \approx 20$.



$\log$ base 5 of 20 is about 1.8614

Change of base for $\log_5 20$.

Logarithms and Logarithmic Functions: frequently asked questions

What does a logarithm mean in plain words?

A logarithm is the exponent that a base must be raised to in order to produce a number. So $\log_3 81 = 4$ because $3^4 = 81$.

Why can you not take the log of a negative number?

A positive base raised to any real power is always positive, so $b^y = x$ never gives $x \leq 0$. That is why the domain of $\log_b x$ is $x > 0$.

What is the difference between $\log$ and $\ln$?

$\log x$ means base 10 and $\ln x$ means base $e \approx 2.718$. Both follow the same properties; only the base differs.

Is $\log(x+3)$ the same as $\log x + \log 3$?

No. The product rule gives $\log 3x = \log 3 + \log x$, but there is no rule for a log of a sum. For example $\log_2(5+3) = 3$, while $\log_2 5 + \log_2 3 \approx 3.907$.

How do you find the inverse of an exponential function?

Swap x and y , then solve for y using a logarithm. The inverse of $f(x) = 2^x + 1$ is $f^{-1}(x) = \log_2(x-1)$.