

Algebra 2 · Function Operations, Composition and Inverses · Worksheet Answer Key

Common Core F-BF.A.1b, F-BF.A.1c, F-BF.B.4a · mathovore.fr

Every solution below is worked step by step, with the final answer in bold. Compare each line with your own work, and when an inverse is found, verify it with a composition or a test point. Mistakes in signs and in the order of composition are the most common errors.

Algebra 2 Common Core: F-BF.A.1b · F-BF.A.1c · F-BF.B.4a

Problem 1: Add and subtract functions

a. $(f+g)(x) = 3x-4+x^2+2x = x^2+5x-4$.

b. $(f-g)(x) = 3x-4-x^2-2x = -x^2+x-4$. Distribute the minus sign to both terms of g .

Problem 2: Multiply functions

a. $(fg)(x) = (x+3)(2x-5) = 2x^2-5x+6x-15 = 2x^2+x-15$.

b. $(fg)(4) = 2(16)+4-15 = 21$. Check: $f(4) \cdot g(4) = 7 \cdot 3 = 21$. **21**

Problem 3: Divide functions and state the domain

$\frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3$. Because $g(3)=0$, $x=3$ is not allowed. $\left(\frac{f}{g}\right)(x) = x+3$, $x \neq 3$; the domain is all real numbers except 3. Check: $\frac{f(5)}{g(5)} = \frac{16}{2} = 8 = 5+3$.

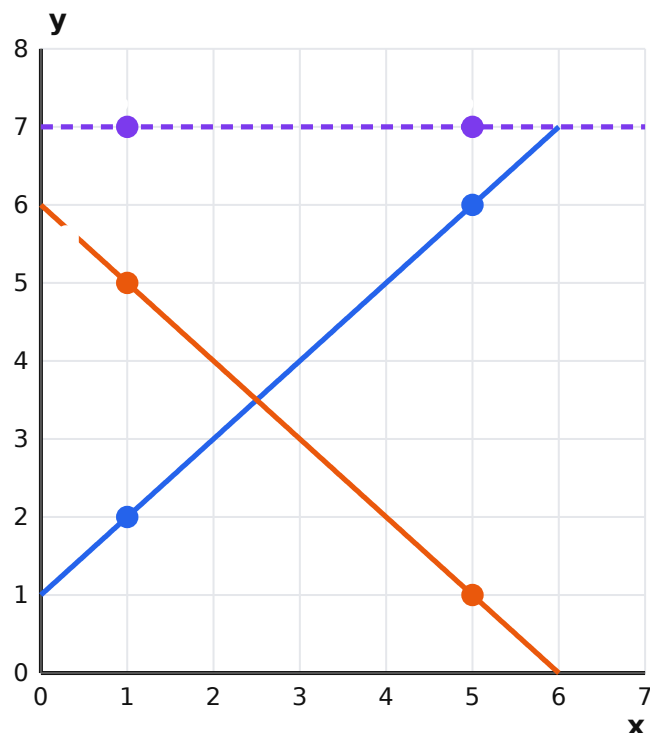
Problem 4: Operations from graphs

Read the values: $f(1)=2$, $g(1)=5$, $f(5)=6$, $g(5)=1$, $f(3)=4$, $g(3)=3$.

a. $(f+g)(1) = 2+5 = \mathbf{7}$

b. $(f-g)(5) = 6-1 = \mathbf{5}$

c. $(fg)(3) = 4 \cdot 3 = \mathbf{12}$



The sum $(f + g)(x)$ is the horizontal line $y = 7$ (violet).

Notice that $f(x) + g(x) = (x+1) + (6-x) = 7$ for every x , so $(f + g)(x)$ is the horizontal line $y = 7$.

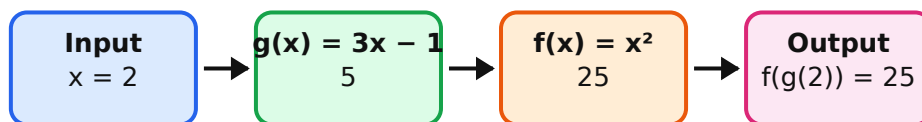
Problem 5: Evaluate a composition

- $g(3) = 10$, so $f(g(3)) = f(10) = 2(10) - 5 = \mathbf{15}$.
- $f(3) = 1$, so $g(f(3)) = g(1) = 1 + 1 = \mathbf{2}$. The order changes the answer.

Problem 6: Write composite functions

- $f(g(x)) = 3x^2 + 4$: $3x^2 + 4$.
- $g(f(x)) = 3(x+4)^2 = 3(x^2 + 8x + 16) = 3x^2 + 24x + 48$.

Problem 7: Trace a value through a machine



$g(2) = 5$ and $f(5) = 25$.

- $g(2) = 3(2) - 1 = 5$ and $f(5) = 25$. $f(g(2)) = 25$.
- $f(g(x)) = (3x-1)^2 = 9x^2 - 6x + 1$. At $x = 2$: $36 - 12 + 1 = 25$, which matches.

Problem 8: Domain of a composition

a. $f(g(x)) = \sqrt{x-5}$ needs $x-5 \geq 0$, so the domain is $x \geq 5$.

b. $g(f(x)) = \sqrt{x-5}$ needs $x \geq 0$, so the domain is $x \geq 0$.

Problem 9: Decompose a function

a. Inner $g(x) = 5x-2$, outer $f(x) = x^3$. Check: $f(g(x)) = (5x-2)^3$.

b. Inner $g(x) = x^2+1$, outer $f(x) = \frac{1}{x}$. Check: $f(g(x)) = \frac{1}{x^2+1}$. Other correct choices exist.

Problem 10: Inverse of a linear function

Write $y = 4x-9$, swap: $x = 4y-9$, so $4y = x+9$ and $f^{-1}(x) = \frac{x+9}{4}$. Check: $f(5) = 11$ and $f^{-1}(11) = \frac{20}{4} = 5$.

Problem 11: Inverse with fractions

Write $y = -\frac{2}{3}x+4$ and swap: $x = -\frac{2}{3}y+4$. Then $x-4 = -\frac{2}{3}y$, so $y = -\frac{3}{2}(x-4) = -\frac{3}{2}x+6$.
 $f^{-1}(x) = -\frac{3}{2}x+6$. Check: $f(3) = 2$ and $f^{-1}(2) = -3+6 = 3$.

Problem 12: Verify inverses by composition

$f(g(x)) = 5 \cdot \frac{x-3}{5} + 3 = x-3+3 = x$ and $g(f(x)) = \frac{(5x+3)-3}{5} = x$. Both compositions give x , so **they are inverses**.

Problem 13: Are they inverses?

$f(g(x)) = 2(\frac{x}{2}-6)+6 = x-12+6 = x-6$, which is not x . **They are not inverses**. The inverse of f :
 $x = 2y+6$, so $f^{-1}(x) = \frac{x-6}{2} = \frac{x}{2}-3$.

Problem 14: Inverse with a restricted domain

Write $x = y^2+3$, so $y^2 = x-3$. Since $y \geq 0$, $f^{-1}(x) = \sqrt{x-3}$, with domain $x \geq 3$ (the range of f).
 $f^{-1}(12) = \sqrt{9} = 3$. Check: $f(3) = 12$.

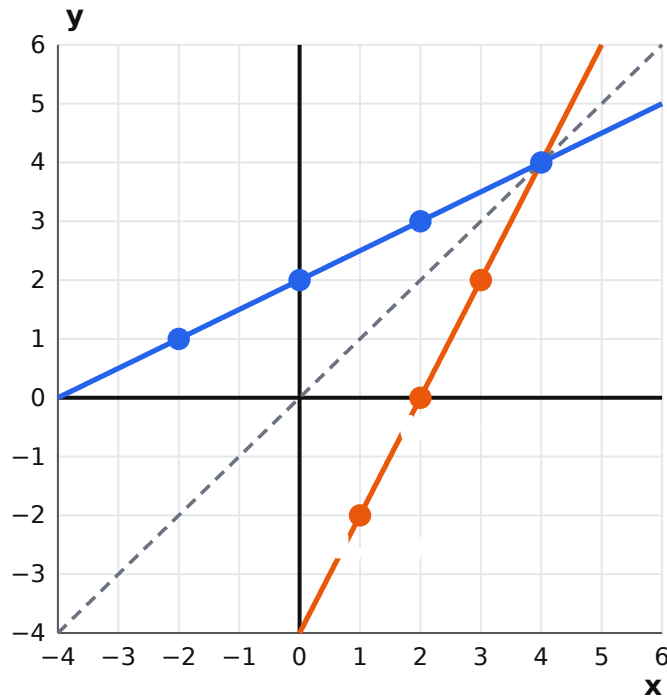
Problem 15: Inverse of a square root function

Write $x = \sqrt{y+2}$. Square both sides: $x^2 = y+2$, so $f^{-1}(x) = x^2-2$. The outputs of f are ≥ 0 , so the domain is $x \geq 0$. Check: $f(7) = \sqrt{9} = 3$ and $f^{-1}(3) = 9-2 = 7$.

Problem 16: Reflect points over $y = x$

a. Swap each pair: $(1,-2)$, $(2,0)$, $(3,2)$, $(4,4)$.

b. The graph of f has slope $\frac{1}{2}$ and y -intercept 2, so $f(x) = \frac{1}{2}x+2$. Swap and solve: $x = \frac{1}{2}y+2$, so $f^{-1}(x) = 2x-4$. It passes through all four swapped points.



The graph of f (blue), its inverse (orange), and the line $y = x$ (dashed).

Problem 17: Horizontal line test

- Yes. At $(0,3)$ and $(4,3)$, since $(0-2)^2-1=3$ and $(4-2)^2-1=3$.
- No.** Two different inputs give the output 3, so f is not one-to-one and its inverse would not be a function.
- Restrict to $x \geq 2$ (or $x \leq 2$). Each horizontal line then meets the graph at most once.

Problem 18: Inverse after restricting the domain

Write $x = (y-2)^2 - 1$, so $(y-2)^2 = x+1$. Because $y \geq 2$, $y-2 = \sqrt{x+1}$ and $f^{-1}(x) = 2 + \sqrt{x+1}$, domain $x \geq -1$. Check: $f(5) = 9 - 1 = 8$ and $f^{-1}(8) = 2 + \sqrt{9} = 5$.

Problem 19: Inverse of a rational function

Write $x = \frac{2y}{y+3}$. Then $xy + 3x = 2y$, so $3x = 2y - xy = y(2-x)$ and $f^{-1}(x) = \frac{3x}{2-x}$, $x \neq 2$. Check: $f(3) = \frac{6}{6} = 1$ and $f^{-1}(1) = \frac{3}{1} = 3$.

Problem 20: Compositions that are equal

$f(g(x)) = 2x^2 - 1$ and $g(f(x)) = (2x-1)^2 = 4x^2 - 4x + 1$. Set equal: $2x^2 - 1 = 4x^2 - 4x + 1$, so $0 = 2x^2 - 4x + 2 = 2(x-1)^2$. $x = 1$ is the only solution. Check: $f(g(1)) = f(1) = 1$ and $g(f(1)) = g(1) = 1$.

Problem 21: Temperature conversion

- a. Write $F = \frac{9}{5}C + 32$ and solve for C : $F - 32 = \frac{9}{5}C$, so $C(F) = \frac{5}{9}(F - 32)$.
- b. $C(95) = \frac{5}{9}(63) = 35$, so $95^\circ\text{F} = 35^\circ\text{C}$. $C(-4) = \frac{5}{9}(-36) = -20$, so $-4^\circ\text{F} = -20^\circ\text{C}$. Both points lie on the graph.

Problem 22: Car rental inverse

- a. Write $C = 0.45m + 25$ and solve for m : $m(C) = \frac{C - 25}{0.45}$.
- b. The inverse takes a bill and returns the number of miles driven. $m(70) = \frac{45}{0.45} = 100$, so the bill of \$70 covers **100 miles**. Check: $0.45(100) + 25 = 70$.

Problem 23: Discount and coupon order

- a. $c(d(x)) = 0.8x - 5$ and $d(c(x)) = 0.8(x - 5) = 0.8x - 4$.
- b. Discount first: $c(d(60)) = 48 - 5$, so **\$43**. Coupon first: $d(c(60)) = 0.8(55)$, so **\$44**.
- c. Taking the 20% off **first** is cheaper. The difference is $(0.8x - 4) - (0.8x - 5) = 1$, so it saves **\$1** at every price.

Problem 24: Composition in context

- a. $A(r(t)) = \pi(3t)^2 = 9\pi t^2$.
- b. $A(r(2)) = 9\pi(4) = 36\pi$, about **113.1 square inches**. Check: $r(2) = 6$ and $\pi \cdot 36 = 36\pi$.