

Algebra 2 · Function Operations, Composition and Inverses · Lesson

Common Core F-BF.A.1b, F-BF.A.1c, F-BF.B.4a · mathovore.fr

An inverse function reverses another function: if f sends a to b , then f inverse sends b back to a . To find it, write $y = f(x)$, swap x and y , and solve for y . Check with composition, because $f(f \text{ inverse}(x))$ and $f \text{ inverse}(f(x))$ must both equal x . The graphs are mirror images across the line $y = x$.

Algebra 2 Common Core: F-BF.A.1b · F-BF.A.1c · F-BF.B.4a

Key facts:

- Composition: $(f \circ g)(x) = f(g(x))$; the inner function g runs first and order usually matters.
- Inverse method: write $y = f(x)$, swap x and y , solve for y , rename it $f^{-1}(x)$.
- Two functions are inverses when $f(g(x)) = x$ and $g(f(x)) = x$ for every x in the domains.
- The graph of f^{-1} is the reflection of the graph of f across the line $y = x$; points (a, b) become (b, a) .
- Only one-to-one functions have inverses; restrict the domain (such as $x \geq 0$ for x^2) when needed.

How do you combine functions with operations and composition?

Functions can be added, subtracted, multiplied, and divided output by output:

$(f+g)(x) = f(x)+g(x)$, $(fg)(x) = f(x) \cdot g(x)$, and $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ where $g(x) \neq 0$. **Composition**, written $f \circ g$, sends the output of one function into another: the value of $f \circ g$ at x is $f(g(x))$.



Composition works like two machines in a row: g runs first, then f . Here $f(g(3)) = 19$.

Order matters

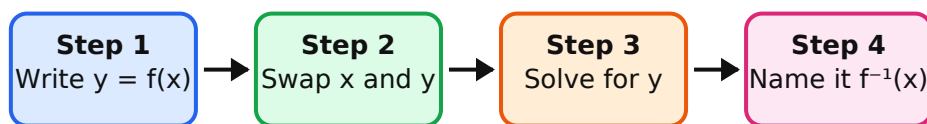
In $f(g(x))$ the inner function g runs first. The domain of the composite is every x in the domain of g for which $g(x)$ is in the domain of f . In general $f(g(x)) \neq g(f(x))$.

For $f(x) = 2x + 3$ and $g(x) = x^2 - 1$:

Combination	Result
$(f+g)(x)$	$x^2 + 2x + 2$
$(f-g)(x)$	$-x^2 + 2x + 4$
$(fg)(x)$	$2x^3 + 3x^2 - 2x - 3$
$f(g(x))$	$2x^2 + 1$
$g(f(x))$	$4x^2 + 12x + 8$

How do you find the inverse of a function?

The **inverse function** f^{-1} undoes f : $f^{-1}(f(x)) = x$ and $f(f^{-1}(x)) = x$. The -1 is a label, not an exponent, so $f^{-1}(x)$ is not $\frac{1}{f(x)}$. Only a one-to-one function, one that passes the horizontal line test, has an inverse function.



The four-step method for finding an inverse function.

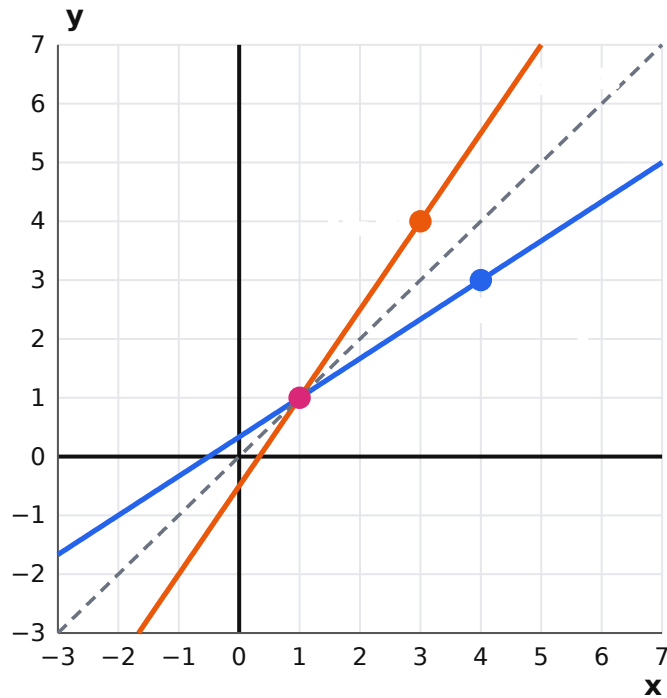
Worked example: find the inverse of $f(x) = \frac{2x+1}{3}$.

1. Write $y = \frac{2x+1}{3}$.
2. Swap the variables: $x = \frac{2y+1}{3}$.
3. Solve for y : $3x = 2y+1$, so $y = \frac{3x-1}{2}$.
4. Name it: $f^{-1}(x) = \frac{3x-1}{2}$.

Check by composition: $f\left(\frac{3x-1}{2}\right) = \frac{(3x-1)+1}{3} = x$.

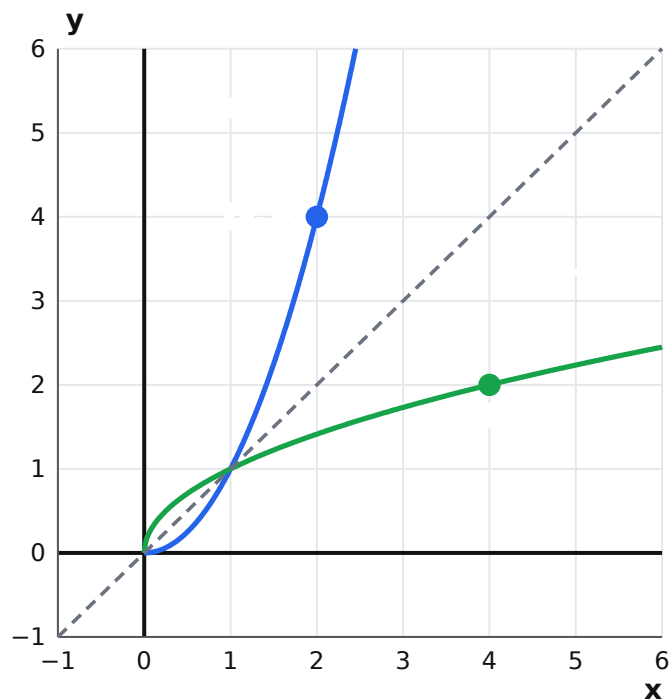
What does the graph of an inverse look like?

Swapping x and y turns each point (a,b) of f into the point (b,a) of f^{-1} . So the two graphs are reflections of each other across the line $y = x$, and the domain of f becomes the range of f^{-1} .



The graph of f (blue) and the graph of f^{-1} (orange) are mirror images across $y = x$ (dashed).

A function that fails the horizontal line test can still be inverted on a restricted domain. The parabola $f(x) = x^2$ is not one-to-one, but on $x \geq 0$ its inverse is $f^{-1}(x) = \sqrt{x}$.



Restricting $f(x) = x^2$ to $x \geq 0$ gives a one-to-one function whose inverse is the square root.

Function Operations, Composition and Inverses: frequently asked

questions

Is $f^{-1}(x)$ the same as $1/f(x)$?

No. The notation f^{-1} means the inverse function, which undoes f . For $f(x) = 2x$ the inverse is $f^{-1}(x) = x/2$, while the reciprocal is $1/(2x)$. These are different functions.

Does the order of composition matter?

Usually yes. For $f(x) = x + 1$ and $g(x) = x^2$, $f(g(2)) = 5$ but $g(f(2)) = 9$. The inner function always runs first.

How can you tell whether a function has an inverse function?

Apply the horizontal line test: if every horizontal line meets the graph at most once, the function is one-to-one and has an inverse. Functions that are always increasing or always decreasing pass the test.

How do you check that two functions are inverses?

Compute $f(g(x))$ and $g(f(x))$. If both simplify to x , they are inverses. For $f(x) = 5x + 3$ and $g(x) = (x - 3)/5$, both compositions give x .

Why do you restrict the domain before finding an inverse?

A function such as $f(x) = x^2$ sends both 2 and -2 to 4, so the inverse would not be a function. Restricting to $x \geq 0$ makes the function one-to-one, and the inverse is $\sqrt{x}$.