

Algebra 2 · Interpreting and Comparing Functions · Lesson

Common Core F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.8a, F-IF.C.9 · mathovore.fr

This algebra 2 function interpretation lesson explains interpreting and comparing functions in one printable page, with definitions, a method and a worked example.

To interpret a function, read its key features in context: intercepts, where it increases or decreases, maximum or minimum, asymptotes and period. The context sets the domain. The average rate of change over an interval is the change in output divided by the change in input. Factoring or completing the square reveals zeros and extreme values of a quadratic.

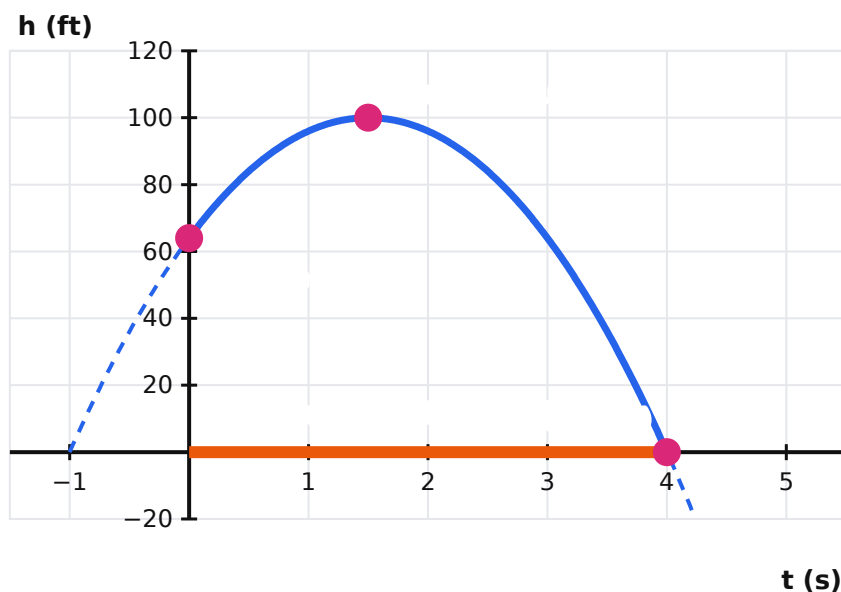
Algebra 2 Common Core: F-IF.B.4 · F-IF.B.5 · F-IF.B.6 · F-IF.C.8a · F-IF.C.9

Algebra 2 Function Interpretation Lesson: key facts

- Standards F-IF.B.4, F-IF.B.5, F-IF.B.6, F-IF.C.8a and F-IF.C.9 on interpreting and comparing functions.
- The domain of a model is limited by the context, such as time in the air or whole numbers of tickets.
- Average rate of change on $[a, b]$ is $\frac{f(b)-f(a)}{b-a}$, the slope of a secant line.
- Factored form $a(x-r)(x-s)$ shows zeros; vertex form $a(x-h)^2+k$ shows the extreme value k .
- Compare two functions by finding the same feature (intercept, maximum, rate of change) from each representation.

What are the key features of a function?

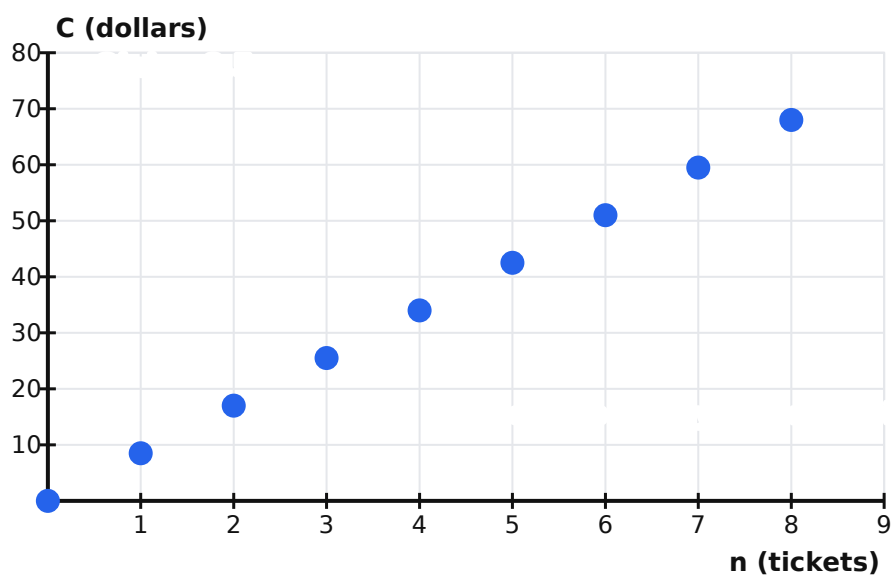
The key features of a graph are its intercepts, the intervals where it increases or decreases, its maximum or minimum, symmetry, end behavior, asymptotes and, for periodic functions, its period. In context each feature has a meaning. A ball is thrown so that its height is $h(t) = -16t^2 + 48t + 64$ feet after t seconds. The intercept 64 is the release height, the vertex $(1.5, 100)$ is the peak, and the zero $t = 4$ is the landing time.



The ball's height graph; only the solid part ($0 \leq t \leq 4$) fits the situation.

How does the context limit the domain?

The **domain** is the set of inputs that make sense in the situation, not every input the formula allows. The ball is in the air only for $0 \leq t \leq 4$, so negative times and times after landing are excluded. Counting situations are discrete. If tickets cost \$8.50 each, then $C(n) = 8.5n$ is defined only for whole numbers n , and its graph is a set of separate points.



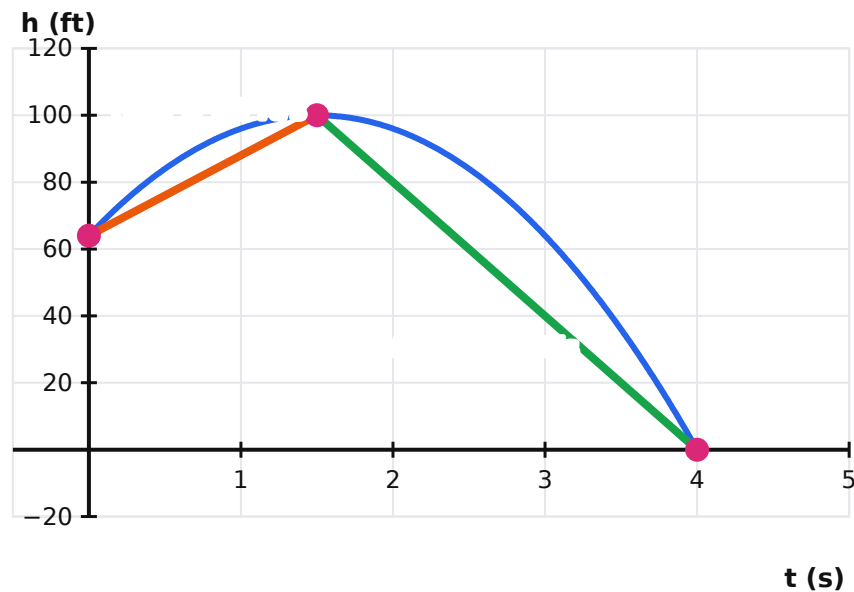
A discrete domain: ticket counts are whole numbers.

How do you find an average rate of change?

Average rate of change

For a function f on the interval $[a, b]$, the average rate of change is $\frac{f(b) - f(a)}{b - a}$. It is the slope of the secant line, measured in output units per input unit.

From $t = 0$ to $t = 1.5$, the height goes from 64 to 100 feet, so the rate is $\frac{100 - 64}{1.5} = 24$ ft/s. From $t = 1.5$ to $t = 4$, it is $\frac{0 - 100}{2.5} = -40$ ft/s: the ball rises, then falls faster.



Each average rate of change is the slope of a secant segment.

How does rewriting a quadratic reveal its features?

Factoring exposes the zeros, and completing the square exposes the vertex. To complete the square for $h(t) = -16t^2 + 48t + 64$:

1. Factor -16 from the t -terms: $-16(t^2 - 3t) + 64$.
2. Add and subtract the square of half of 3, which is 2.25, inside the parentheses.
3. Group the perfect square and bring -2.25 outside as $+36$: $-16(t - 1.5)^2 + 36 + 64$.
4. Combine: $h(t) = -16(t - 1.5)^2 + 100$.

Standard form

$$h = -16t^2 + 48t + 64$$

shows the y-intercept 64

Factored form

$$h = -16(t - 4)(t + 1)$$

shows the zeros $t = 4$ and $t = -1$

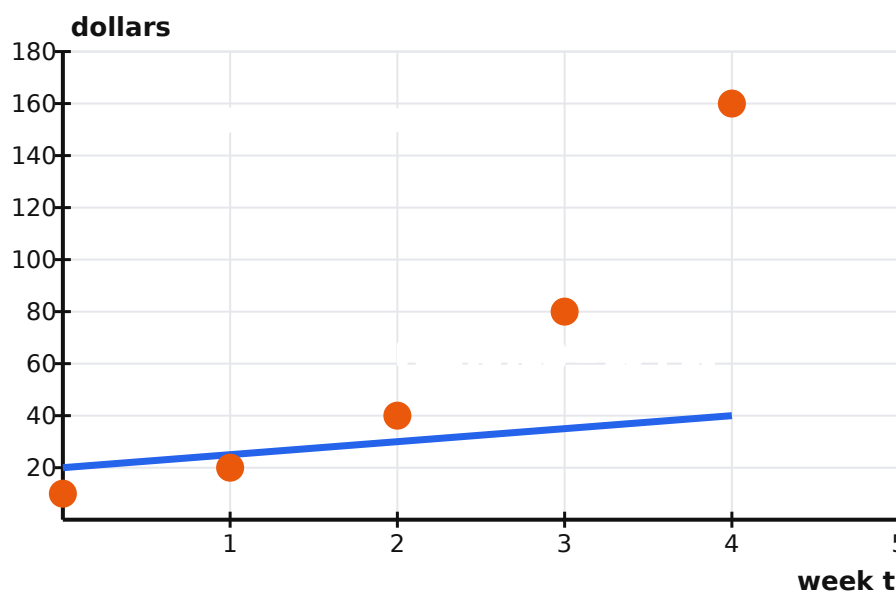
Vertex form

$$h = -16(t - 1.5)^2 + 100$$

shows the maximum 100 at $t = 1.5$

How do you compare two functions given in different ways?

Find the same property for each function, whatever its representation. Plan A saves money by the equation $A(t) = 20 + 5t$ dollars after t weeks. Plan B starts at \\$10 and doubles each week, with values 10, 20, 40, 80, 160. At $t = 4$, A has \\$40 and B has \\$160. On $[0, 4]$, A's average rate is 5 dollars per week, while B's is $\frac{160 - 10}{4} = 37.5$ dollars per week.



Plan A (line) and Plan B (points) compared on the same axes.

Interpreting and Comparing Functions: frequently asked questions

What are the key features of a graph?

Intercepts, intervals of increase and decrease, relative maximum and minimum values, symmetry, end behavior, asymptotes, and the period of a periodic function. In a model, each feature has a meaning in the situation.

How is the domain of a model different from the domain of its formula?

The formula $h(t) = -16t^2 + 48t + 64$ accepts any real t , but the model makes sense only from release to landing, $0 \leq t \leq 4$. The context decides which inputs count.

What does an average rate of change tell you?

It tells how fast the output changes per unit of input over a whole interval, such as 24 feet per second between $t = 0$ and $t = 1.5$. It equals the slope of the line through the two endpoints of the graph.

When should you factor and when should you complete the square?

Factor a quadratic to read its zeros directly. Complete the square to read the vertex, which gives the maximum or minimum value and the axis of symmetry. The two forms describe the same function.

How do you compare a function given by a table with one given by an equation?

Compute the same quantity for both, such as the value at a chosen input, the maximum, or the average rate of change on the same interval. Then compare the numbers and state which function is greater.

How to use this algebra 2 function interpretation lesson

Use this algebra 2 function interpretation lesson before the worksheet, then check what you learned with the quiz.

