

# Algebra 2 · Structure of Expressions and Modeling with Equations · Worksheet Answer Key

Common Core A-SSE.A.1, A-SSE.A.2, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.D.11, A-APR.C.5, N-CN.C.8, N-CN.C.9 · mathovore.fr

This algebra 2 expressions worksheet answer key solves all 22 problems of the structure of expressions and modeling with equations worksheet step by step.

This answer key shows a full worked solution for each of the 22 problems on expressions and modeling. Compare every step, not only the final answer, and re-check any solution that fails the constraints of the situation.

**Algebra 2 Common Core: A-SSE.A.1 · A-SSE.A.2 · A-CED.A.1 · A-CED.A.2 · A-CED.A.3 · A-CED.A.4 · A-REI.D.11 · A-APR.C.5 · N-CN.C.8 · N-CN.C.9**

## Algebra 2 Expressions Worksheet Answer Key: problems 1-5

### Problem 1: Parts of an exponential model

- The starting amount is the factor 800, so Priya opened the account with **\$800**.
- The growth factor is **1.03**. Since  $1.03 = 1 + 0.03$ , the balance grows **3%** each year.
- When  $t = 0$ ,  $(1.03)^0 = 1$  so  $B = 800$ : it is the opening deposit, **\$800**.

### Problem 2: Terms and coefficients

- The terms are  $5x^3$ ,  $-2x^2$ ,  $7x$  and  $-9$ .
- The coefficients are **5, -2 and 7**.
- The constant term is **-9**.
- The factors are **3,  $(x+1)$  and  $(x-4)$** .

### Problem 3: Meaning of a factor

- The factor  $p$  is the price per shirt, and  $500 - 20p$  is the number of shirts sold.
- At  $p = 5$ ,  $500 - 20(5) = 400$  shirts sell, so  $R = 5(400) = 2000$ : **\$2,000**.
- At  $p = 10$ ,  $500 - 20(10) = 300$  shirts sell, so  $R = 10(300) = 3000$ : **\$3,000**.
- The \$10 price earns \$1,000 more than the \$5 price, so **\$10** is better.

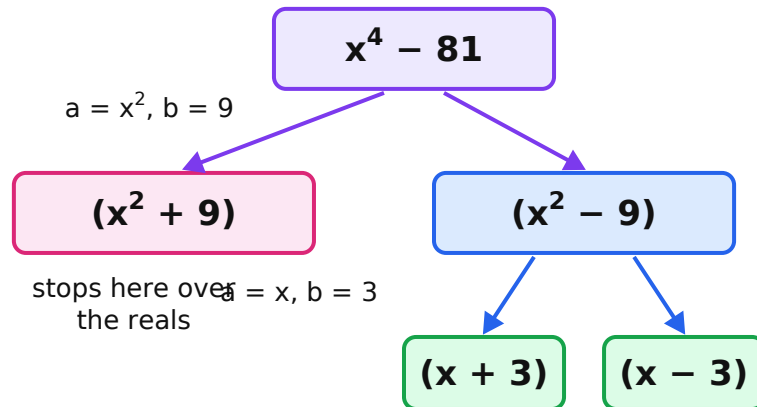
### Problem 4: Write and interpret a cost expression

- Notebooks cost  $2.5n$  and the backpack costs 18, so the total is  $1.08(2.5n + 18)$ .

- b. The factor 1.08 is  $1+0.08$ : it multiplies the entire sum by the 8% tax.  
 c.  $1.08(2.5 \cdot 6 + 18) = 1.08(33) = 35.64$ , so the cost is **\$35.64**.  
 d.  $1.08(2.5n) = 2.7n$  and  $1.08(18) = 19.44$ , so the cost is  $2.7n + 19.44$ .

### Problem 5: Fourth-power differences

- a.  $x^4 - 16 = (x^2 + 4)(x^2 - 4)$ , so  $(x^2 + 4)(x + 2)(x - 2)$ .  
 b.  $x^4 - 81 = (x^2 + 9)(x^2 - 9)$ , so  $(x^2 + 9)(x + 3)(x - 3)$ .  
 c.  $16x^4 - 1 = (4x^2 + 1)(4x^2 - 1)$ , so  $(4x^2 + 1)(2x + 1)(2x - 1)$ .  
 d.  $m^4 - n^4 = (m^2 + n^2)(m^2 - n^2)$ , so  $(m^2 + n^2)(m + n)(m - n)$ .



Part (b): two rounds of the difference-of-squares pattern.

## Algebra 2 Expressions Worksheet Answer Key: problems 6-10

### Problem 6: Hidden squares

- a. With  $a = x + 3$  and  $b = 4$ :  $(x + 3 - 4)(x + 3 + 4) = (x - 1)(x + 7)$ .  
 b. With  $a = 2x - 1$  and  $b = 5$ :  $(2x - 6)(2x + 4) = 4(x - 3)(x + 2)$ .  
 c.  $x^4 - 5x^2 + 4 = (x^2 - 1)(x^2 - 4)$ , so  $(x - 1)(x + 1)(x - 2)(x + 2)$ .  
 d.  $x^4 - 13x^2 + 36 = (x^2 - 4)(x^2 - 9)$ , because  $4 + 9 = 13$  and  $4 \cdot 9 = 36$ . The result is  $(x - 2)(x + 2)(x - 3)(x + 3)$ .

### Problem 7: Simplify with structure

- a.  $x^4 - 16 = (x^2 + 4)(x^2 - 4)$ , so the quotient is  $x^2 - 4$ . The denominator  $x^2 + 4$  is never 0, so there are no excluded values.  
 b.  $\frac{(x-7)(x+7)}{(x+7)^2} = \frac{x-7}{x+7}$ ,  $x \neq -7$ .  
 c.  $x^4 - 1 = (x^2 + 1)(x^2 - 1)$ , so the quotient is  $x^2 + 1$  with  $x \neq 1$  and  $x \neq -1$ .

### Problem 8: Factor over the complex numbers

- a.  $x^2 - (3i)^2$  since  $(3i)^2 = -9$ :  $(x + 3i)(x - 3i)$ .  
 b.  $(5i)^2 = -25$ , so  $(x + 5i)(x - 5i)$ .  
 c.  $(2x)^2 - i^2 = 4x^2 + 1$ , so  $(2x + i)(2x - i)$ .

d.  $x^4 - 16 = (x+2)(x-2)(x^2+4)$  and  $x^2+4 = (x+2i)(x-2i)$ , so  $(x+2)(x-2)(x+2i)(x-2i)$ .

### Problem 9: Quadratic zeros and the Fundamental Theorem

a.  $x = \frac{6 \pm \sqrt{36-52}}{2} = \frac{6 \pm 4i}{2} = 3 \pm 2i$ . Factored:  $(x-3-2i)(x-3+2i)$ .

b.  $x = \frac{-2 \pm \sqrt{4-40}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$ . Factored:  $(x+1-3i)(x+1+3i)$ .

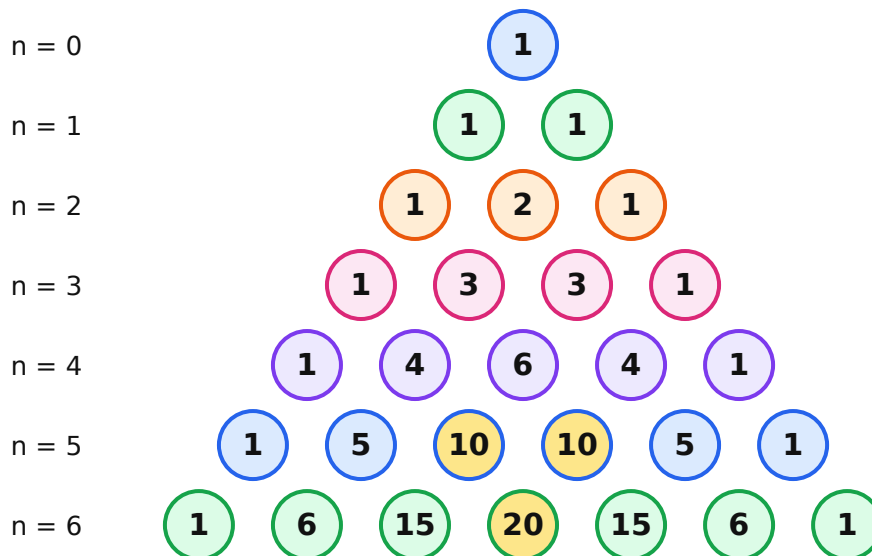
Every quadratic has **exactly two** complex zeros, counting a repeated zero twice.

### Problem 10: Pascal's triangle rows

a. Each number is the sum of the two above it. Row 5 is **1, 5, 10, 10, 5, 1** and row 6 is **1, 6, 15, 20, 15, 6, 1**.

b. The coefficients of  $(a+b)^5$  come from row 5, in order for  $a^5, a^4b, a^3b^2$ , so the coefficient is **10**.

c.  $1+6+15+20+15+6+1=64$ , which is  $2^6$ . Each row sums to a power of 2: **64**.



Row 6 built from row 5:  $10 + 10 = 20$ .

## Algebra 2 Expressions Worksheet Answer Key: problems 11-15

### Problem 11: Expand binomials

a. Row 4 is 1, 4, 6, 4, 1:  $x^4 + 4x^3(2) + 6x^2(4) + 4x(8) + 16 = x^4 + 8x^3 + 24x^2 + 32x + 16$ .

b. Row 3 is 1, 3, 3, 1:  $x^3 + 3x^2(-3) + 3x(9) + (-27) = x^3 - 9x^2 + 27x - 27$ .

c. Row 5 is 1, 5, 10, 10, 5, 1 with  $a=2x$  and  $b=-1$ :  $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$ . Check at  $x=1$ :  $32-80+80-40+10-1=1=1^5$ . The expansion is  $32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1$ .

### Problem 12: Rearrange common formulas

a. Multiply by 2, then divide by  $h$ :  $b = \frac{2A}{h}$ .

b.  $2w = P - 2l$ , so  $w = \frac{P-2l}{2}$ .

c. Divide both sides by  $\pi r^2$ :  $h = \frac{V}{\pi r^2}$ .

d.  $F - 32 = \frac{9}{5}C$ , so  $C = \frac{5}{9}(F - 32)$ .

### Problem 13: Rearrange formulas with fractions

a.  $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{fp}$ , so  $q = \frac{fp}{p-f}$ .

b.  $S(1-r) = a$  gives  $1-r = \frac{a}{S}$ , so  $r = 1 - \frac{a}{S} = \frac{S-a}{S}$ .

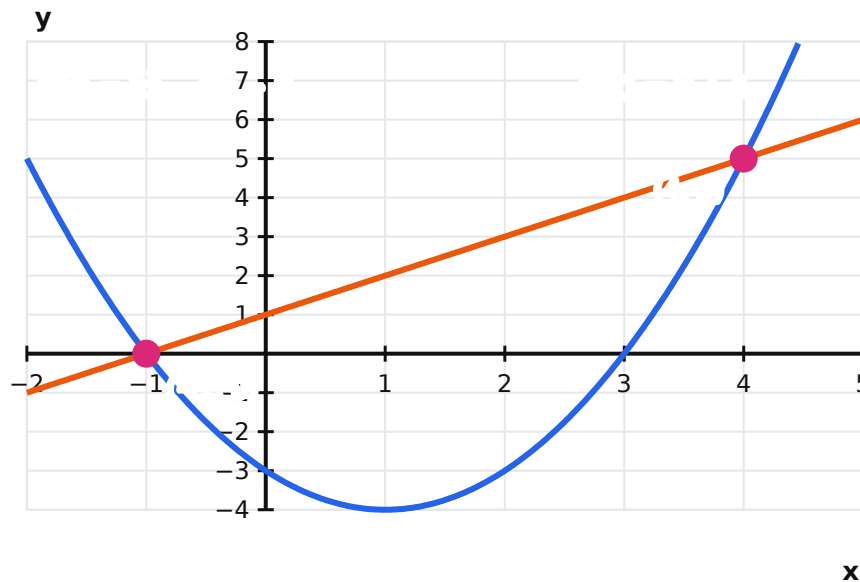
c. Divide by  $(1+r)^t$ :  $P = \frac{A}{(1+r)^t}$ .

d.  $y(x-3) = 2x+1$ , then  $xy-3y = 2x+1$ , then  $x(y-2) = 3y+1$ . So  $x = \frac{3y+1}{y-2}$ . (Check:  $x=4$  gives  $y=9$  and  $\frac{28}{7} = 4$ .)

### Problem 14: Solve $f(x) = g(x)$ from a graph

a. The graphs cross at  $x = -1$  and  $x = 4$ .

b.  $x^2 - 2x - 3 = x + 1$  gives  $x^2 - 3x - 4 = 0$ , so  $(x-4)(x+1) = 0$ . The solutions are  $x = -1$  and  $x = 4$ . At  $x = 4$  both functions equal 5; at  $x = -1$  both equal 0.



The crossing points are  $(-1, 0)$  and  $(4, 5)$ .

### Problem 15: Solve $f(x) = g(x)$ from a table

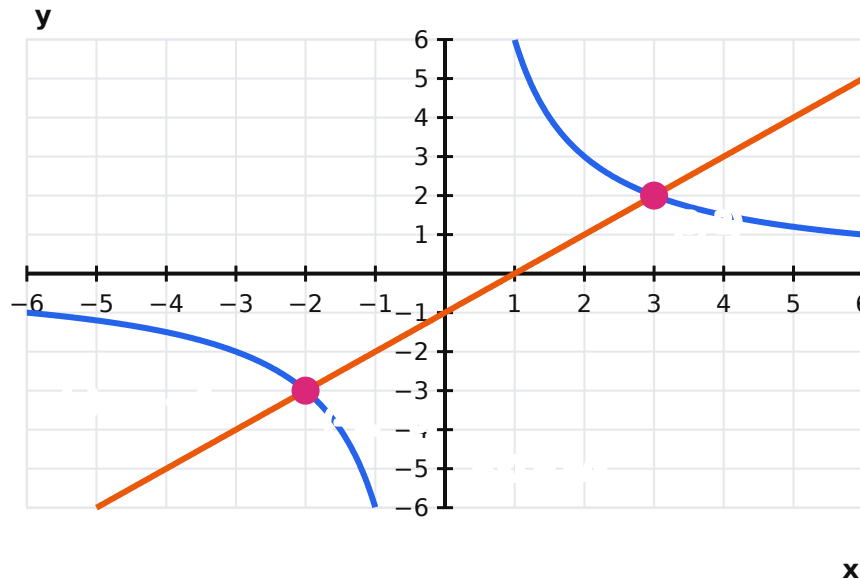
| x    | 1  | 2 | 3 | 4  | 5  |
|------|----|---|---|----|----|
| f(x) | 2  | 4 | 8 | 16 | 32 |
| g(x) | 10 | 9 | 8 | 7  | 6  |

The two rows match at  $x = 3$  where both equal 8, so the solution is  $x = 3$ . The function  $f$  always increases while  $g$  always decreases, so they can meet only once.

## Algebra 2 Expressions Worksheet Answer Key: problems 16-20

### Problem 16: Solve a rational equation by graphing

- a. The graphs cross near  $x=3$  and  $x=-2$ .
- b. Multiply both sides by  $x$ :  $6 = x(x-1)$ , so  $x^2 - x - 6 = 0$  and  $(x-3)(x+2) = 0$ . The solutions are  $x=3$  **and**  $x=-2$ . Check:  $\frac{6}{3} = 2 = 3-1$  and  $\frac{6}{-2} = -3 = -2-1$ .



The crossing points are  $(3, 2)$  and  $(-2, -3)$ .

### Problem 17: Garden area model

- a.  $w(w+3) = 270$ , or  $w^2 + 3w - 270 = 0$ .
- b.  $(w+18)(w-15) = 0$  gives  $w = -18$  or  $w = 15$ . A width cannot be negative, so reject  $-18$ . The garden is **15 ft by 18 ft**, and  $15 \cdot 18 = 270$ .
- c. The perimeter is  $2(15+18) = 66$  ft, and  $66 \cdot 4.50 = 297$ . The fence costs **\$297**.

### Problem 18: Speed and time model

- a. Time is distance divided by speed:  $\frac{60}{r} + \frac{60}{r+20} = 2.5$ .
- b. Multiply by  $r(r+20)$ :  $60(r+20) + 60r = 2.5r(r+20)$ , so  $120r + 1200 = 2.5r^2 + 50r$  and  $2.5r^2 - 70r - 1200 = 0$ . Dividing by 2.5 gives  $r^2 - 28r - 480 = 0$ , so  $(r-40)(r+12) = 0$ . A speed cannot be negative, so  $r = 40$ .
- c. The country road speed is **40 mph** and the interstate speed is **60 mph**. Check:  $1.5 + 1 = 2.5$  hours.

### Problem 19: Bacteria growth model

- a. Doubling every 3 hours gives  $N(t) = 500 \cdot 2^{t/3}$ .
- b.  $N(6) = 500 \cdot 2^2 = 2000$  bacteria.
- c.  $500 \cdot 2^{t/3} = 16000$  gives  $2^{t/3} = 32 = 2^5$ , so  $t/3 = 5$  and  $t = 15$  **hours**.

## Problem 20: Budget inequality

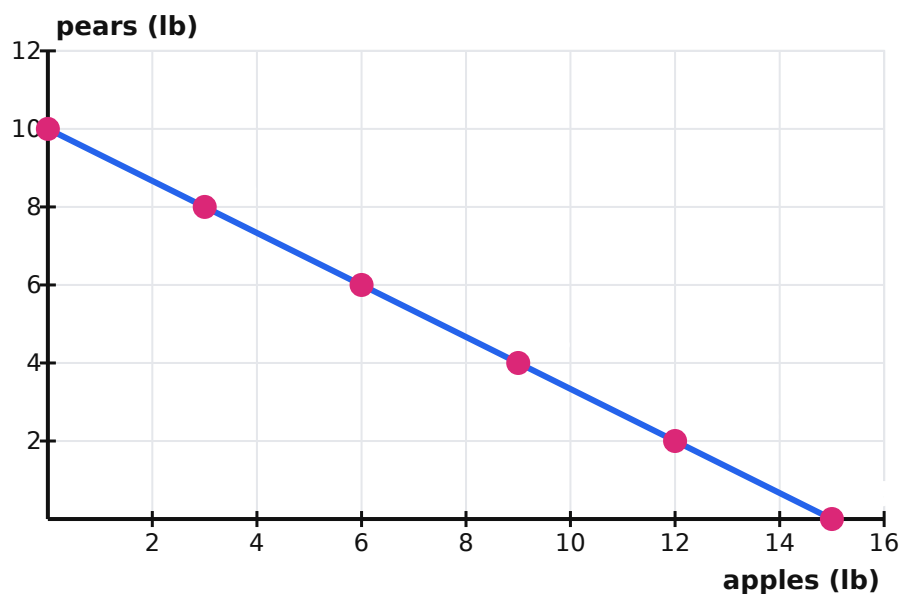
- a.  $30 + 6.5n \leq 120$ .
- b.  $6.5n \leq 90$ , so  $n \leq 13.85$  (rounded).
- c. Shirts come in whole numbers, so  $n$  must be a nonnegative integer. The greatest value is **13 shirts**: they cost  $30 + 6.5(13) = 114.50$ , but 14 shirts cost \$121.

## Algebra 2 Expressions Worksheet Answer Key: problems 21-22

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### Problem 21: Farmers market equation

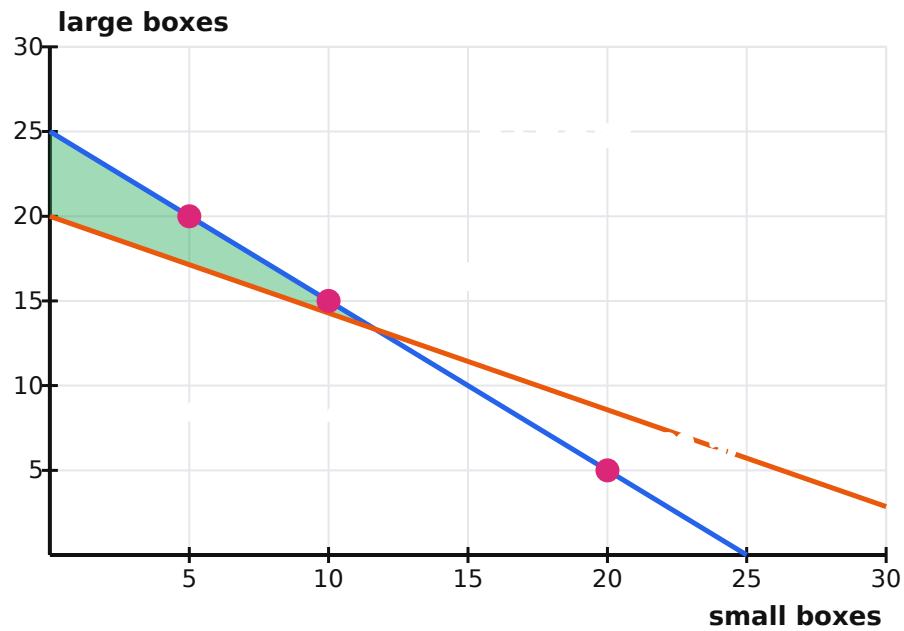
- a.  $2a + 3p = 30$ .
- b. Since  $p = \frac{30 - 2a}{3}$  must be a whole number,  $a$  must be a multiple of 3. The pairs are **(0, 10), (3, 8), (6, 6), (9, 4), (12, 2), (15, 0)**. Check (6, 6):  $12 + 18 = 30$ .
- c. Neither amount can be negative:  $a \geq 0$  and  $p \geq 0$ .



The six viable purchases.

### Problem 22: Cookie box constraints

- a.  $x + y \leq 25$ ,  $4x + 7y \geq 140$ ,  $x \geq 0$ ,  $y \geq 0$ .
- b. A:  $10 + 15 = 25$  and  $40 + 105 = 145$  work. B:  $5 + 20 = 25$  and  $20 + 140 = 160$  work. C:  $20 + 5 = 25$  but  $80 + 35 = 115 < 140$  fails. So **A and B are possible**.
- c. With  $x = 25 - y$ :  $4(25 - y) + 7y \geq 140$  gives  $100 + 3y \geq 140$ , so  $y \geq 13.33$ . Whole boxes force  $y = 14$ :  $4(11) + 7(14) = 142$ , while 13 large boxes give  $4(12) + 7(13) = 139$ .



The shaded triangle holds every possible plan; A and B lie on its edge, C does not.

## How to use this algebra 2 expressions worksheet answer key

Read this algebra 2 expressions worksheet answer key after trying each problem yourself, and redo any problem you missed.

