

Algebra 2 · Structure of Expressions and Modeling with Equations · Lesson

Common Core A-SSE.A.1, A-SSE.A.2, A-CED.A.1, A-CED.A.2, A-CED.A.3, A-CED.A.4, A-REI.D.11, A-APR.C.5, N-CN.C.8, N-CN.C.9 · mathovore.fr

This algebra 2 expressions equations models lesson explains structure of expressions and modeling with equations in one printable page, with definitions, a method and a worked example.

The structure of an expression shows how its parts relate: terms are added, factors are multiplied, and patterns such as a difference of squares allow rewriting. Algebra 2 uses that structure to factor, to model situations with equations and inequalities, to rearrange formulas, to find where two functions are equal, and to expand powers with Pascal's triangle.

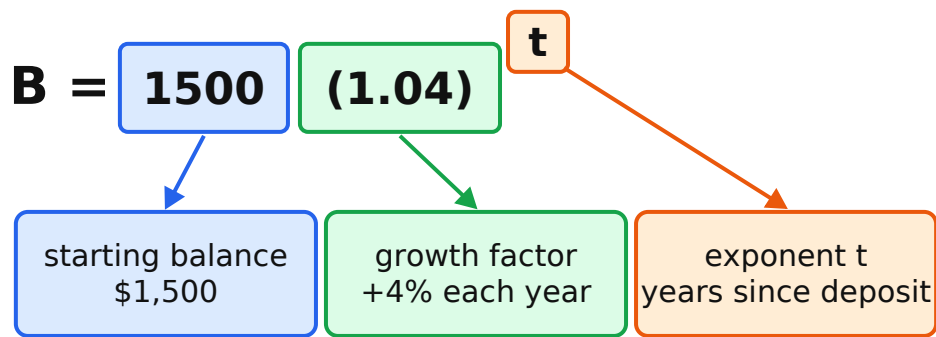
Algebra 2 Common Core: A-SSE.A.1 · A-SSE.A.2 · A-CED.A.1 · A-CED.A.2 · A-CED.A.3 · A-CED.A.4 · A-REI.D.11 · A-APR.C.5 · N-CN.C.8 · N-CN.C.9

Algebra 2 Expressions Equations Models Lesson: key facts

- Standards A-SSE.A.1, A-SSE.A.2, A-CED.A.1-4, A-REI.D.11, A-APR.C.5 and N-CN.C.8-9 on expressions and modeling.
- A difference of squares can hide inside higher powers: $x^4 - y^4 = (x^2 + y^2)(x + y)(x - y)$.
- The solutions of $f(x) = g(x)$ are the x -coordinates where the two graphs cross.
- Row n of Pascal's triangle gives the coefficients of $(a+b)^n$.
- Over the complex numbers, $x^2 + 4 = (x + 2i)(x - 2i)$, and every quadratic has exactly two zeros.

What does the structure of an expression tell us?

An expression is built from **terms** that are added and **factors** that are multiplied. Reading that structure reveals the story behind it. In the savings model $B = 1500(1.04)^t$, the factor 1500 is the starting balance in dollars, 1.04 is the growth factor (a 4% raise each year), and t counts years.



Each part of the model $B = 1500(1.04)^t$ has a meaning in the savings story.

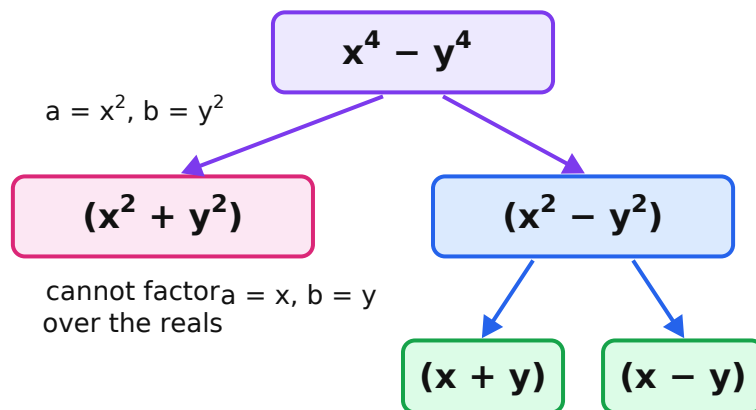
Key vocabulary

Term: a piece separated by plus or minus signs. **Factor:** a piece that is multiplied.

Coefficient: the numerical factor of a term, such as 3 in $3x^2$.

How can you see a hidden pattern and rewrite an expression?

The pattern $a^2 - b^2 = (a+b)(a-b)$ works even when a and b are whole expressions. For $x^4 - y^4$, let $a = x^2$ and $b = y^2$. Then $x^4 - y^4 = (x^2 + y^2)(x^2 - y^2)$, and the second factor is a difference of squares again.



Two rounds of the difference-of-squares pattern split the fourth-power difference.

How do you build an equation or inequality from a situation?

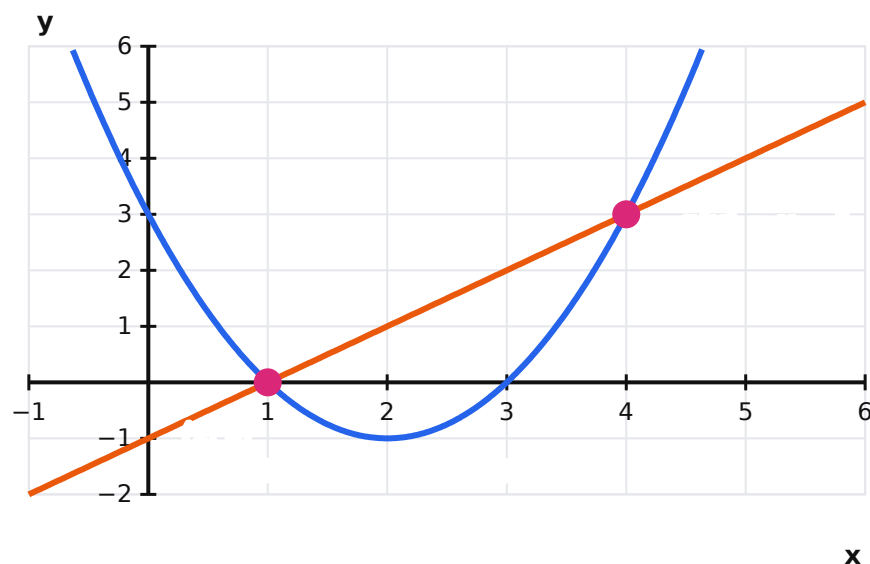
1. Name each variable and its unit.
2. Translate each relationship into an expression.
3. Set expressions equal, or compare them with an inequality.
4. Record constraints, such as whole numbers or nonnegative values.
5. Solve, then check the answer against the constraints.

Example. A rectangular pen is 3 feet longer than it is wide and has an area of 40 square feet. With width w feet, $w(w+3)=40$, so $w^2+3w-40=0$ and $(w+8)(w-5)=0$. The constraint $w>0$ rejects $w=-8$, so the pen is 5 ft by 8 ft.

To rearrange a formula, isolate one variable as you would in any equation. From $C=\frac{5}{9}(F-32)$, multiply by $\frac{9}{5}$ and add 32 to get $F=\frac{9}{5}C+32$.

How do you solve $f(x) = g(x)$ with a graph or a table?

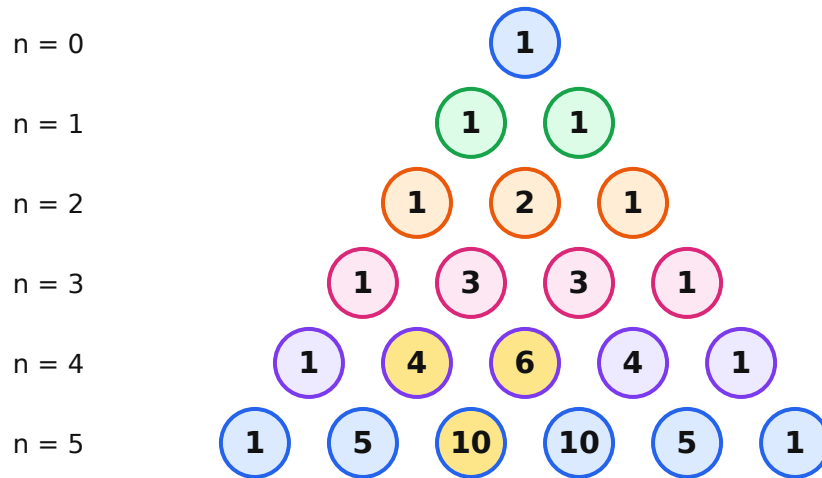
Graph both functions. The x -coordinates of the crossing points are the solutions. Below, $f(x)=x^2-4x+3$ and $g(x)=x-1$ meet at $x=1$ and $x=4$. A table confirms it, because both functions equal 0 at $x=1$ and equal 3 at $x=4$.



The graphs of f and g cross where $f(x) = g(x)$: at $x = 1$ and $x = 4$.

What is the Binomial Theorem?

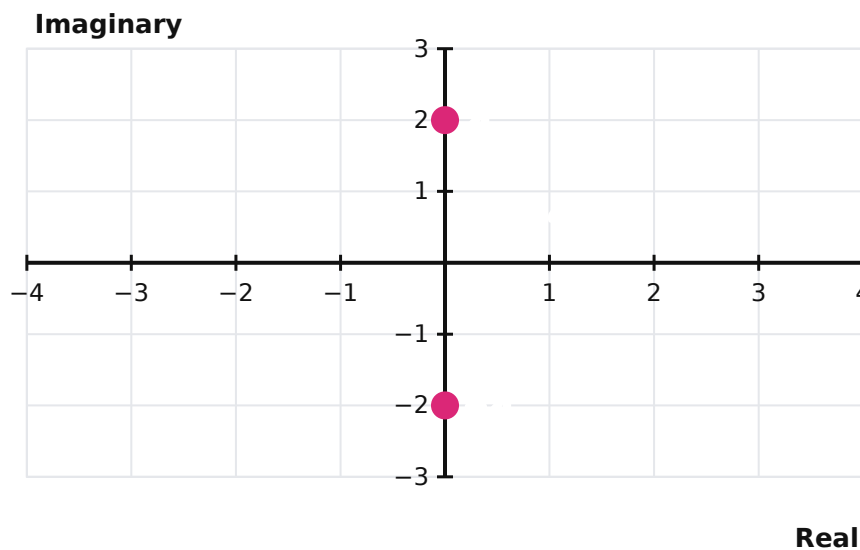
Row n of Pascal's triangle lists the coefficients of $(a+b)^n$. Row 4 is 1, 4, 6, 4, 1, so $(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$.



Each entry is the sum of the two above it: $4 + 6 = 10$.

How do complex numbers finish the factoring?

Because $i^2 = -1$, we get $(x+2i)(x-2i) = x^2 - 4i^2 = x^2 + 4$. The Fundamental Theorem of Algebra says a polynomial of degree n has exactly n complex zeros, counting repeats, so every quadratic has two.



The two zeros of $x^2 + 4$ are $2i$ and $-2i$.

Structure of Expressions and Modeling with Equations: frequently asked questions

What is the difference between a term and a factor?

Terms are added or subtracted, and factors are multiplied. In $3x^2+5$, the terms are $3x^2$ and 5 . In $3x^2$, the factors are 3 , x and x .

How do you factor $x^4 - y^4$?

Treat it as $(x^2)^2 - (y^2)^2$ and apply the difference of squares to get $(x^2 + y^2)(x^2 - y^2)$. Then factor $x^2 - y^2$ as $(x + y)(x - y)$. The factor $x^2 + y^2$ stays because it has no real factors.

Why do models need constraints?

A constraint says which solutions make sense in the story. A width cannot be negative, and a number of shirts must be a whole number, so some algebraic solutions are rejected.

How does Pascal's triangle help expand a binomial?

Row n supplies the coefficients in order. For $(x + 2)^3$, row 3 is 1, 3, 3, 1, so the expansion is $x^3 + 3x^2(2) + 3x(2)^2 + 2^3 = x^3 + 6x^2 + 12x + 8$.

How many zeros does a quadratic have?

Exactly two complex zeros, counting a repeated zero twice. The zeros of $x^2 + 4$ are $2i$ and $-2i$, and the zeros of $(x - 3)^2$ are 3 and 3.

How to use this algebra 2 expressions equations models lesson

Use this algebra 2 expressions equations models lesson before the worksheet, then check what you learned with the quiz.

