

Algebra 2 · Exponential Functions and the Number e · Worksheet Answer Key

Common Core F-IF.C.8b, F-LE.A.4, F-LE.B.5 · mathovore.fr

Each solution shows the model, the substitution, and the rounded result in bold. Small differences in the last decimal can come from rounding too early, so keep full precision until the final step. Check every logarithm answer by substituting it back into the original equation.

Algebra 2 Common Core: F-IF.C.8b · F-LE.A.4 · F-LE.B.5

Problem 1: Growth or decay?

- a. $a=800$, $b=1.06$: **growth of 6%** per period.
- b. $a=500$, $b=0.92$: **decay of 8%** (since $1-0.92=0.08$).
- c. $a=3$, $b=1.5$: **growth of 50%**.

Problem 2: Population model

- a. $P(t)=25000(1.04)^t$, because $b=1+0.04$.
- b. $P(5)=25000(1.04)^5 \approx \mathbf{30,416}$ and $P(10)=25000(1.04)^{10} \approx \mathbf{37,006}$.

Problem 3: Depreciation model

- a. $V(t)=28000(0.85)^t$, since $b=1-0.15=0.85$.
- b. $V(4)=28000(0.85)^4 \approx \mathbf{\$14,616.17}$.

Problem 4: Evaluate powers of e

- a. $e^2 \approx \mathbf{7.389}$
- b. $e^{-1} = \frac{1}{e} \approx \mathbf{0.368}$
- c. $e^{0.5} = \sqrt{e} \approx \mathbf{1.649}$

Problem 5: Annual compounding

$A=1200(1.05)^8$, so the balance is **\$1,772.95**.

Problem 6: Quarterly compounding

$A=3000\left(1+\frac{0.06}{4}\right)^{4 \cdot 5} = 3000(1.015)^{20}$, so the balance is **\$4,040.57**.

Problem 7: Monthly compounding

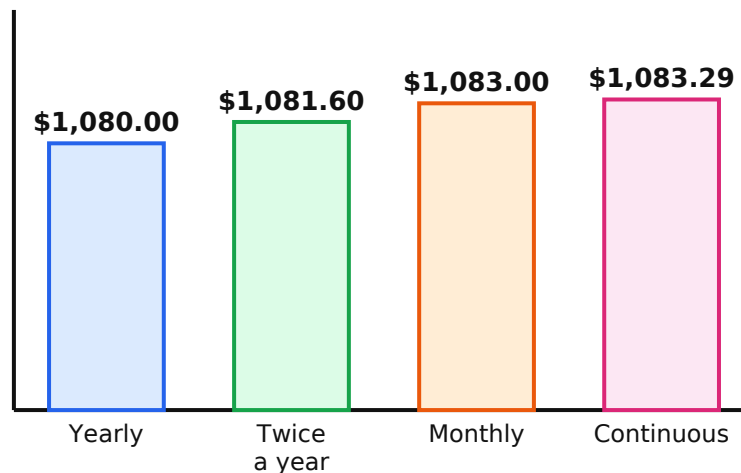
$A = 5000\left(1 + \frac{0.036}{12}\right)^{120} = 5000(1.003)^{120}$, so the balance is **\$7,162.79**.

Problem 8: Continuous compounding

$A = 4000e^{0.05 \cdot 7} = 4000e^{0.35}$, so the balance is **\$5,676.27**.

Problem 9: Compare compounding methods

- a. Yearly: $1000(1.08) = \mathbf{\$1,080.00}$. Twice a year: $1000(1.04)^2 = \mathbf{\$1,081.60}$. Monthly: $1000\left(1 + \frac{0.08}{12}\right)^{12} = \mathbf{\$1,083.00}$. Continuous: $1000e^{0.08} = \mathbf{\$1,083.29}$.
- b. $\$1,083.29 - \$1,080.00 = \mathbf{\$3.29}$ (using unrounded balances).



Values after 1 year. The axis starts at \$1,060.

Problem 10: Interpret the parameters

- a. The account starts with **\$1,500** and grows **8% per year** since $1.08 = 1 + 0.08$. Each year the balance is multiplied by 1.08.
- b. The dose starts at **240 mg/L** and the concentration **falls 15% each hour**, since $0.85 = 1 - 0.15$.

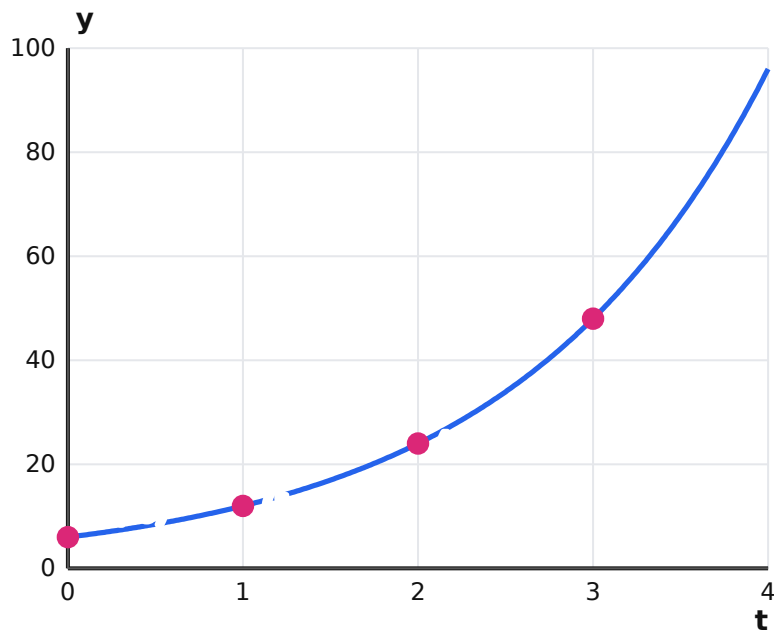
Problem 11: Find the rate from data

Let b be the yearly factor. Then $400b^2 = 441$, so $b^2 = \frac{441}{400}$ and $b = \frac{21}{20} = 1.05$. The rate is **5% per year**. Check: $400(1.05)^2 = 441$.

Problem 12: Model from two points

- a. $f(0) = \alpha = 6$. Then $6b^3 = 48$, so $b^3 = 8$ and $b = 2$ (so $\alpha = 6$).

b. $f(t) = 6 \cdot 2^t$, and $f(5) = 6 \cdot 32 = \mathbf{192}$.



The curve $f(t) = 6 \cdot 2^t$.

Problem 13: Equivalent forms of a rate

a. $1.08^t = (1.08^{1/12})^{12t}$, so $b = 1.08^{1/12} \approx \mathbf{1.00643}$.

b. The monthly rate is $b - 1 \approx \mathbf{0.64\%}$ per month. This is less than $\frac{8\%}{12} \approx 0.67\%$ because interest on interest builds up.

Problem 14: Solve with logarithms

Divide by 3: $2^t = \frac{50}{3}$. Take the natural log: $t \ln 2 = \ln \frac{50}{3}$. So $t = \frac{\ln(50/3)}{\ln 2} \approx \mathbf{4.06}$.

Problem 15: Time to reach a goal

$1.05^t = \frac{5}{3}$, so $t = \frac{\ln(5/3)}{\ln 1.05} \approx \mathbf{10.47 \text{ years}}$. Check: $1200(1.05)^{10.47} \approx 2000$.

Problem 16: Doubling time

Solve $1.04^t = 2$: $t = \frac{\ln 2}{\ln 1.04} \approx \mathbf{17.67 \text{ years}}$. The starting amount does not matter, because $a \cdot 1.04^t = 2a$ reduces to $1.04^t = 2$.

Problem 17: Half-life model

a. $A(18) = 80\left(\frac{1}{2}\right)^3 = 80 \cdot \frac{1}{8} = \mathbf{10 \text{ mg}}$.

b. $A(15) = 80\left(\frac{1}{2}\right)^{2.5} \approx \mathbf{14.14 \text{ mg}}$. This lies between 20 mg (12 h) and 10 mg (18 h).

Problem 18: Decay time with logs

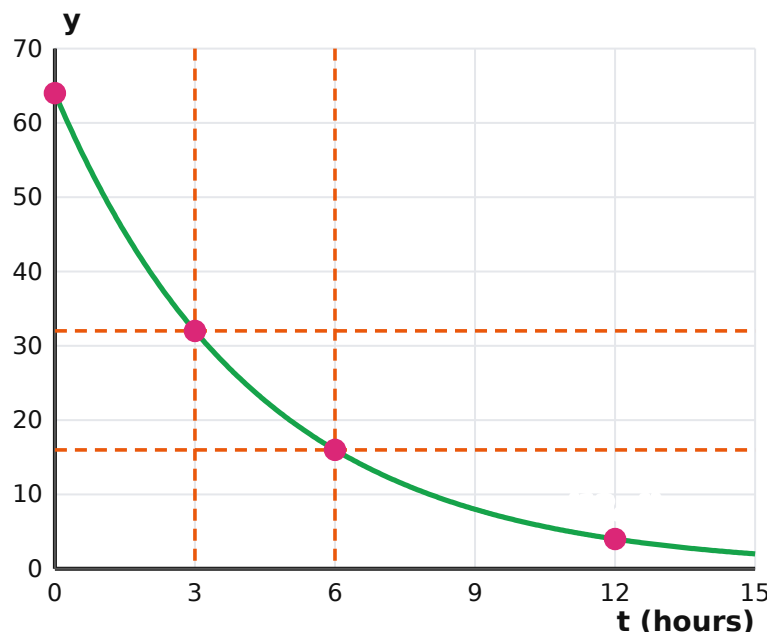
Solve $100\left(\frac{1}{2}\right)^{t/12} = 20$: $\left(\frac{1}{2}\right)^{t/12} = \frac{1}{5}$, so $\frac{t}{12}\ln\frac{1}{2} = \ln\frac{1}{5}$ and $\frac{t}{12} = \frac{\ln 5}{\ln 2}$. Then $t = 12 \cdot \frac{\ln 5}{\ln 2} \approx \mathbf{27.86 \text{ days}}$.

Problem 19: Continuous growth model

- a. $500e^{6k} = 2000$, so $e^{6k} = 4$ and $k = \frac{\ln 4}{6} \approx \mathbf{0.2310}$.
b. $N(10) = 500e^{10k} = 500 \cdot 4^{10/6} \approx \mathbf{5,040}$ bacteria.

Problem 20: Read a half-life from a graph

- a. The amount drops from 64 to 32 between $t=0$ and $t=3$, so the half-life is **3 hours**.
b. $A(t) = 64\left(\frac{1}{2}\right)^{t/3}$.
c. $A(15) = 64\left(\frac{1}{2}\right)^5 = \mathbf{2 \text{ mg}}$. For 1 mg, $\left(\frac{1}{2}\right)^{t/3} = \frac{1}{64} = \left(\frac{1}{2}\right)^6$, so $t = \mathbf{18 \text{ hours}}$.



The amount halves every 3 hours (dashed lines).

Problem 21: Compare two accounts

Diego: $2000\left(1 + \frac{0.052}{12}\right)^{120} \approx \mathbf{\$3,360.28}$. Priya: $2000e^{0.0525 \cdot 10} = 2000e^{0.525} \approx \mathbf{\$3,380.92}$. Priya has more, by \$20.64.

Problem 22: Tripling time

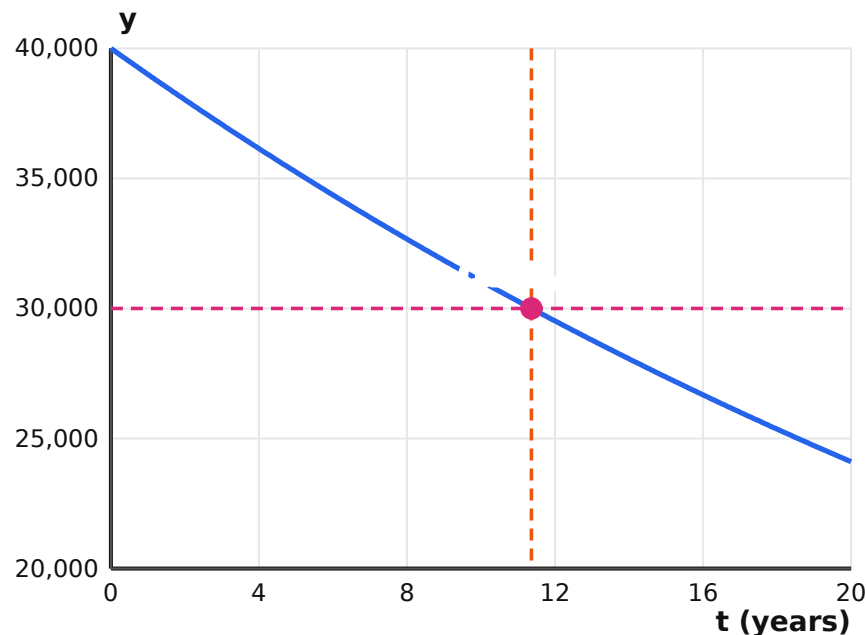
Solve $Pe^{0.06t} = 3P$: $e^{0.06t} = 3$, so $0.06t = \ln 3$ and $t = \frac{\ln 3}{0.06} \approx \mathbf{18.31 \text{ years}}$.

Problem 23: Savings goal

Solve $6000(1.00375)^{12t} = 10000$: $(1.00375)^{12t} = \frac{5}{3}$, so $12t \ln 1.00375 = \ln \frac{5}{3}$. Then $t = \frac{\ln(5/3)}{12 \ln 1.00375} \approx$
11.37 years. Check: $6000(1.00375)^{12(11.35)} \approx 10000$.

Problem 24: Shrinking town

- a. $P(t) = 40000(0.975)^t$.
 b. Solve $40000(0.975)^t = 30000$: $0.975^t = 0.75$, so $t = \frac{\ln 0.75}{\ln 0.975} \approx 11.36$. Check: $P(11) \approx 30,277$ is still above 30,000, and $P(12) \approx 29,520$ is below. The answer is **12 years**.



The curve crosses $P = 30,000$ at about $t = 11.36$.