

Algebra 2 · Exponential Functions and the Number e · Lesson

Common Core F-IF.C.8b, F-LE.A.4, F-LE.B.5 · mathovore.fr

An exponential model $y = a \cdot b^t$ multiplies a starting amount a by a constant factor b each period, with $b = 1 + r$ for growth and $b = 1 - r$ for decay. Compound interest uses $A = P(1 + r/n)^{nt}$, and continuous compounding uses $A = Pe^{rt}$, where e is about 2.71828. Logarithms solve for time.

Algebra 2 Common Core: F-IF.C.8b · F-LE.A.4 · F-LE.B.5

Key facts:

- $y = a \cdot b^t$: a is the initial value and b is the factor per period.
- Percent rate r gives $b = 1 + r$ for growth and $b = 1 - r$ for decay.
- Compound interest: $A = P(1 + r/n)^{nt}$; continuous: $A = Pe^{rt}$ with $e \approx 2.71828$.
- Half-life model: $A = A_0(1/2)^{t/h}$; doubling time at rate r is $\ln 2 \div \ln(1 + r)$.
- To solve $a \cdot b^t = d$ for t , isolate b^t and take a logarithm: $t = \ln(d/a) \div \ln b$.

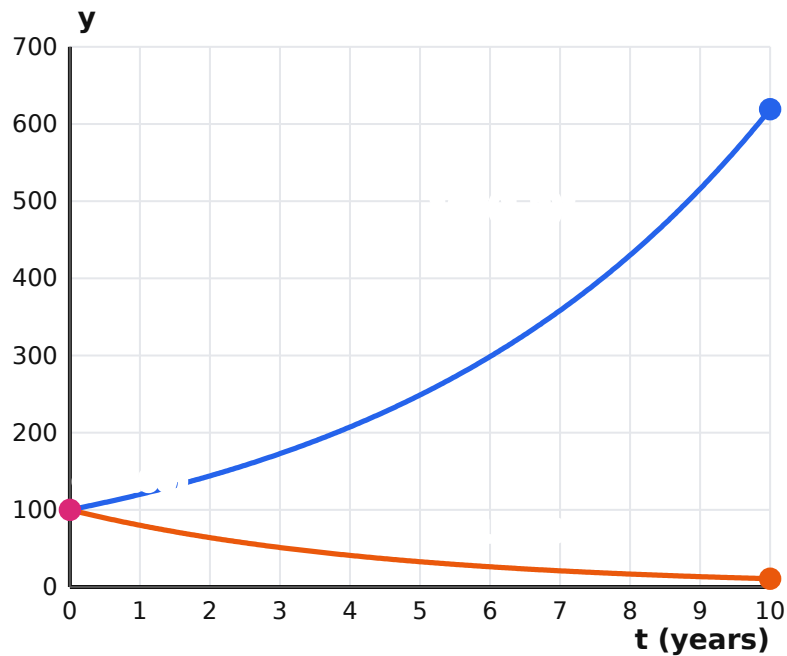
What are exponential growth and decay?

An **exponential model** has the form $y = a \cdot b^t$. The value a is the starting amount, and the **growth factor** b multiplies the amount in every period. A rate r written as a decimal gives $b = 1 + r$ for growth and $b = 1 - r$ for decay.



Reading the parts of $y = a \cdot b^t$. Growth: $b = 1 + r$. Decay: $b = 1 - r$.

Example: a town of 12,000 people grows 3.5% each year, so $P(t) = 12000(1.035)^t$. After 10 years $P(10) = 12000(1.035)^{10}$, which is about **16,927 people**.



Growth (blue, $b = 1.2$) and decay (orange, $b = 0.8$) both start at 100.

Vocabulary

Initial value: the amount at $t = 0$, which is the y -intercept $(0, a)$.

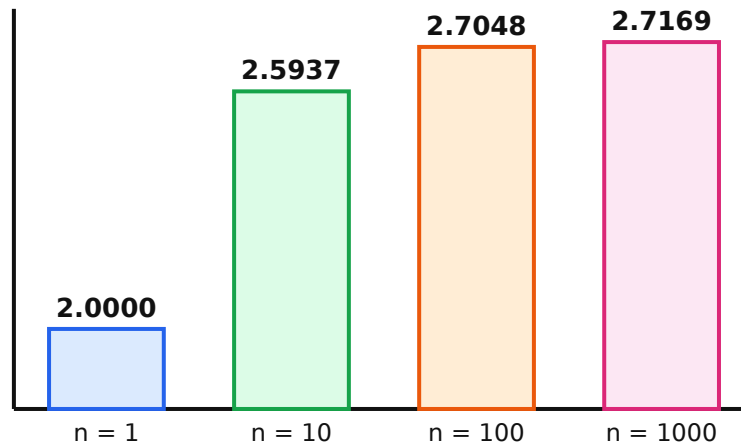
Growth or decay factor: the number b that multiplies the amount each period. If $b > 1$ the model grows, and if $0 < b < 1$ it decays.

Rate: the percent change per period, $r = |b - 1|$.

Changing the time unit changes the factor but not the story. A yearly factor of 1.08 equals a monthly factor of $1.08^{1/12}$, about 1.0064, because twelve months must multiply to 1.08. Every graph of $y = a \cdot b^t$ crosses the y -axis at a and never touches the t -axis, which is a horizontal asymptote.

How does compound interest lead to the number e ?

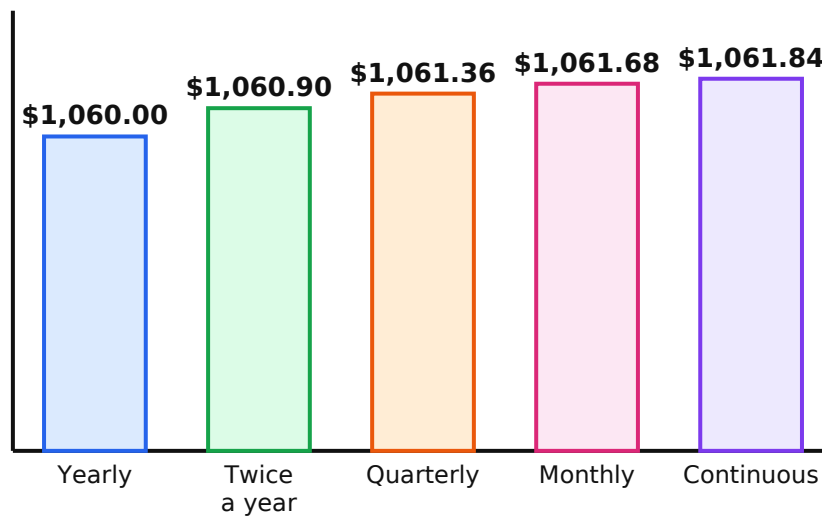
With principal P , annual rate r , and n compounding periods per year, the balance after t years is $A = P \left(1 + \frac{r}{n}\right)^{nt}$. As n grows, the balance approaches a limit, and the factor $\left(1 + \frac{1}{n}\right)^n$ approaches the irrational number $e \approx 2.71828$. Compounding continuously gives $A = P e^{rt}$.



The values $(1 + 1/n)^n$ approach $e \approx 2.71828$ as n grows. The axis starts at 1.8.

Example

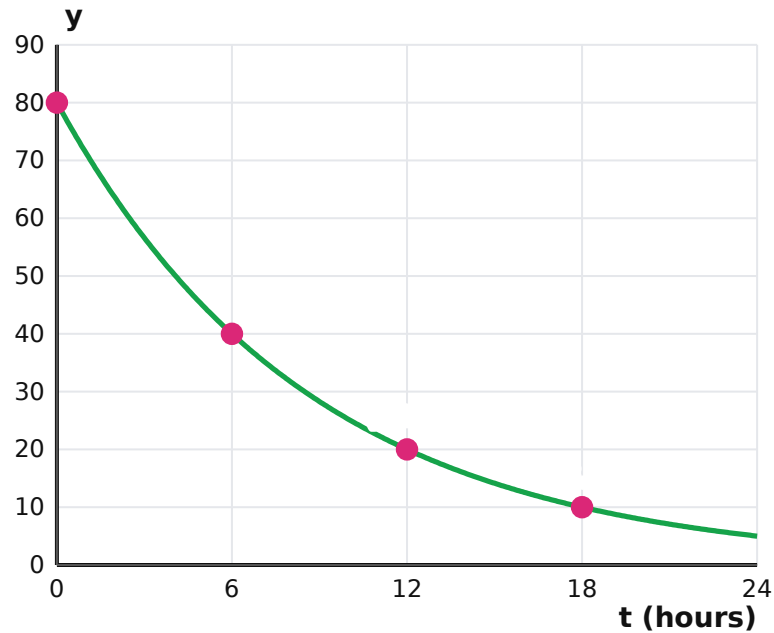
Invest \$2,500 at 4% for 6 years. Quarterly: $A = 2500(1.01)^{24}$ gives **\$3,174.34**. Continuous: $A = 2500e^{0.24}$ gives **\$3,178.12**.



Value after 1 year of \$1,000 at 6%. The bar axis starts at \$1,050.

How do half-life and doubling time work?

A quantity with half-life h follows $A(t) = A_0 \left(\frac{1}{2}\right)^{t/h}$. A sample of 80 mg with $h = 6$ hours has $A(18) = 80 \left(\frac{1}{2}\right)^3 = 10$ mg. To find an unknown time, isolate the power and take a logarithm: the equation $1.05^t = 2$ gives $t = \frac{\ln 2}{\ln 1.05}$, so the **doubling time** at 5% growth is about **14.21 years**.



A sample with half-life 6 hours halves every 6 hours.

Exponential Functions and the Number e: frequently asked questions

How do you turn a percent rate into a growth factor?

Write the rate as a decimal and add it for growth or subtract it for decay. A 6% increase gives $b = 1.06$, and a 15% decrease gives $b = 0.85$.

What is the number e?

The number e is about 2.71828. It is the limit of $(1 + 1/n)^n$ as n grows without bound, and it is the base used for continuous growth, as in $A = Pe^{(rt)}$.

Does compounding more often always earn more?

Yes, but the gain shrinks quickly. At 6% on \$1,000 for one year, yearly compounding earns \$60.00 and monthly compounding earns \$61.68, while continuous compounding earns only \$61.84.

How do you solve for time in an exponential equation?

Isolate the exponential expression, take the natural log of both sides, and use the power rule. For $1200(1.05)^t = 2000$, divide to get $1.05^t = 5/3$, so $t = \ln(5/3) \div \ln(1.05)$.

What is the difference between half-life and doubling time?

Half-life is the time for a decaying quantity to fall to half, and doubling time is the time for a

growing quantity to reach twice its value. Both use the same idea: the amount changes by a fixed factor over a fixed time.