

Algebra 2 · Solving Exponential and Logarithmic Equations · Worksheet Answer Key

Common Core F-LE.A.4, A-SSE.B.3c · mathovore.fr

Each solution shows the isolating, logging and checking steps. If your decimal differs in the last digit, keep more digits in your calculator until the final step.

Algebra 2 Common Core: F-LE.A.4 · A-SSE.B.3c

Problem 1: Same base

- a. $2^5 = 32$, so $x = 5$
- b. $3^4 = 81$, so $x = 4$
- c. $5^{-3} = \frac{1}{125}$, so $x = -3$
- d. $2^{2x} = 2^3$, so $2x = 3$ and $x = \frac{3}{2}$

Problem 2: Rewrite to a common base

- a. $2^{3x-1} = 2^6$, so $3x-1 = 6$ and $x = \frac{7}{3}$
- b. $3^{2x} = 3^{3x-3}$, so $2x = 3x-3$ and $x = 3$
- c. $2^{-2x-2} = 2^4$, so $-2x-2 = 4$ and $x = -3$
- d. $2^{3x} = 2^{-5}$, so $3x = -5$ and $x = -\frac{5}{3}$

Problem 3: Logarithmic to exponential form

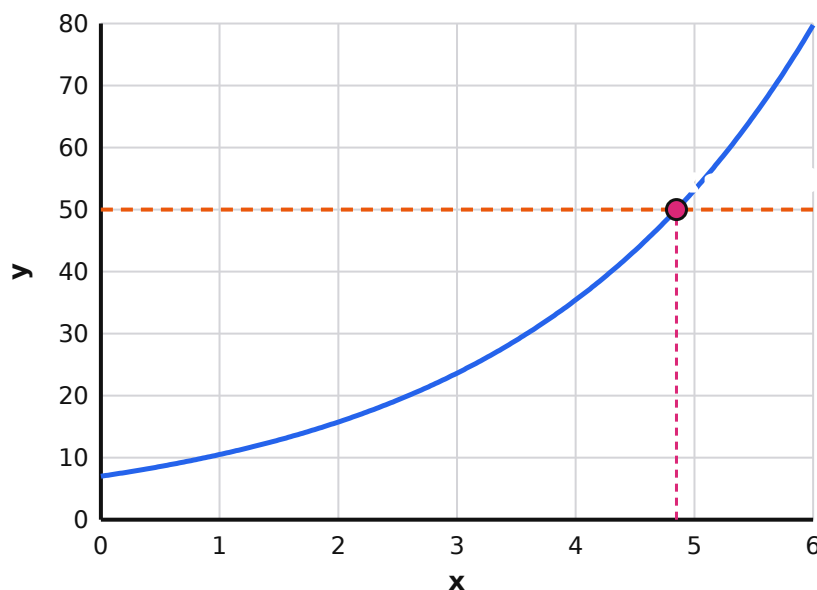
- a. $x = 6^2 = 36$
- b. $x = 2^{-4} = \frac{1}{16}$
- c. $x = 4^{3/2} = (4^{1/2})^3 = 2^3 = 8$
- d. $x = e^3 \approx 20.086$

Problem 4: One-step exponential equations

- a. $x = \frac{\ln 20}{\ln 3} \approx 2.7268$
- b. $x = \frac{\ln 100}{\ln 2} \approx 6.6439$
- c. $x = \log_4 5 \approx 1.6532$
- d. $x = \ln 12 \approx 2.4849$

Problem 5: Isolate the power first

- a. $5^x = 25$, so $x = 2$
b. $3 \cdot 2^x = 48$, $2^x = 16$, so $x = 4$
c. $1.5^x = \frac{50}{7}$, so $x = \frac{\ln(50/7)}{\ln 1.5} \approx \frac{1.9661}{0.4055} \approx \mathbf{4.849}$



The intersection matches the solution.

Problem 6: Exponents with a coefficient

- a. $3x = \frac{\ln 50}{\ln 2} \approx 5.6439$, so $x \approx 1.881$
b. $2x - 1 = \frac{\ln 40}{\ln 5} \approx 2.2920$, $2x \approx 3.2920$, so $x \approx 1.646$

Problem 7: Equations with base e

- a. $e^{2x} = 15$, $2x = \ln 15$, so $x \approx 1.3540$
b. $5e^{0.3x} = 20$, $e^{0.3x} = 4$, $0.3x = \ln 4$, so $x = \frac{\ln 4}{0.3} \approx 4.6210$

Problem 8: Solve a formula for time

- a. $\frac{A}{P} = e^{rt}$, so $\ln \frac{A}{P} = rt$ and $t = \frac{\ln(A/P)}{r}$. With the values: $\frac{\ln 1.750.05}{\text{approx}} \mathbf{11.19}$ years.
b. $\frac{A}{P} = (1+r)^t$, so $t = \frac{\ln(A/P)}{\ln(1+r)}$. With the values: $\frac{\ln 1.75 \ln 1.05}{\text{approx}} \mathbf{11.47}$ years.
 $\frac{\ln 0.55960.0488}{\text{approx}}$

Problem 9: Simple logarithmic equations

- a. $x + 5 = 2^4 = 16$, so $x = 11$
b. $2x - 1 = 3^2 = 9$, so $x = 5$

- c. $x-4=10^1$, so $x=14$
 d. $3x=e^2$, so $x=\frac{e^2}{3} \approx 2.463$

Problem 10: Isolate the logarithm

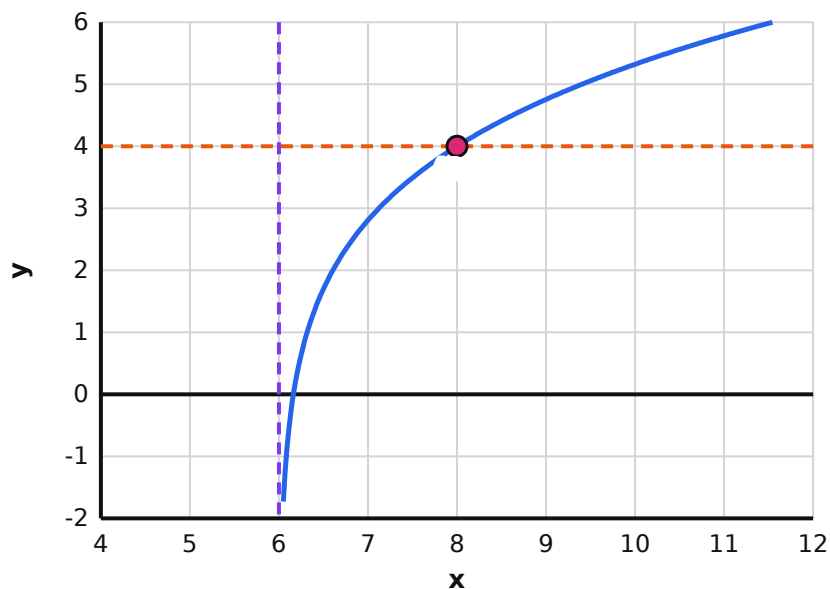
- a. $3\log_2 x=12$, $\log_2 x=4$, so $x=16$
 b. $\ln(x+1)=3$, $x+1=e^3$, so $x=e^3-1 \approx 19.086$
 c. $4\log_5(x-2)=8$, $\log_5(x-2)=2$, $x-2=25$, so $x=27$

Problem 11: Equal logarithms

- a. $3x+1=x+9$, $2x=8$, so $x=4$. Check: 13 and 13 are both positive.
 b. $2x-3=x+5$, so $x=8$. Check: 13 and 13.
 c. $x^2=5x-6$, $x^2-5x+6=0$, $(x-2)(x-3)=0$. Both work: $x=2$ gives 4 and 4; $x=3$ gives 9 and 9. $x=2$ or $x=3$

Problem 12: Add two logarithms

- a. $\log_2(x(x-6))=4$, so $x^2-6x=16$ and $x^2-6x-16=0$. Then $(x-8)(x+2)=0$. $x=-2$ is extraneous because $\log_2(-2)$ is undefined. $x=8$. Check: $3+1=4$.
 b. $x(x+6)=27$, $x^2+6x-27=0$, $(x+9)(x-3)=0$. $x=-9$ is extraneous. $x=3$. Check: $1+2=3$.



The only intersection is at $x=8$.

Problem 13: Subtract two logarithms

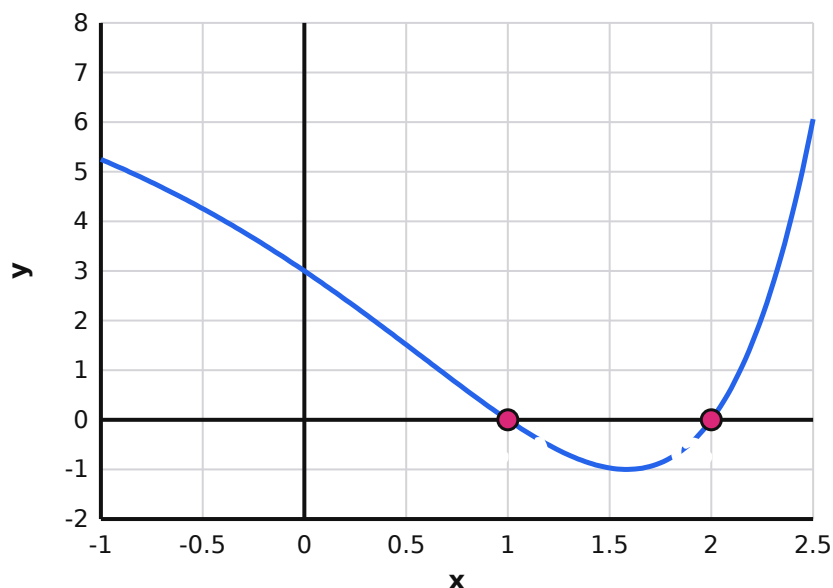
- a. $\log_3 \frac{x+8}{x}=2$, so $\frac{x+8}{x}=9$, $x+8=9x$, and $x=1$. Check: $\log_3 9 - \log_3 1=2$.
 b. $\frac{x+5}{x-1}=3$, $x+5=3x-3$, $2x=8$, so $x=4$. Check: $\ln 9 - \ln 3 = \ln 3$.

Problem 14: Reject extraneous solutions

- a. $x(x-3)=10$, $x^2-3x-10=0$, $(x-5)(x+2)=0$. $x=5$; $x=-2$ is extraneous.
b. $(x+2)(x-1)=4$, $x^2+x-6=0$, $(x+3)(x-2)=0$. $x=2$; $x=-3$ is extraneous.

Problem 15: Quadratic form

Since $2^{2x}=u^2$, the equation is $u^2-6u+8=0$, so $(u-2)(u-4)=0$ and $u=2$ or $u=4$. Then $2^x=2$ gives $x=1$ and $2^x=4$ gives $x=2$. The solutions are $x=1$ **and** $x=2$, the x-intercepts of the graph.



The x-intercepts at $x=1$ and $x=2$.

Problem 16: Quadratic form with e

Let $u=e^x$: $u^2-5u+6=0$, $(u-2)(u-3)=0$. Then $e^x=2$ and $e^x=3$, so $x=\ln 2 \approx 0.6931$ or $x=\ln 3 \approx 1.0986$.

Problem 17: Equivalent growth rates

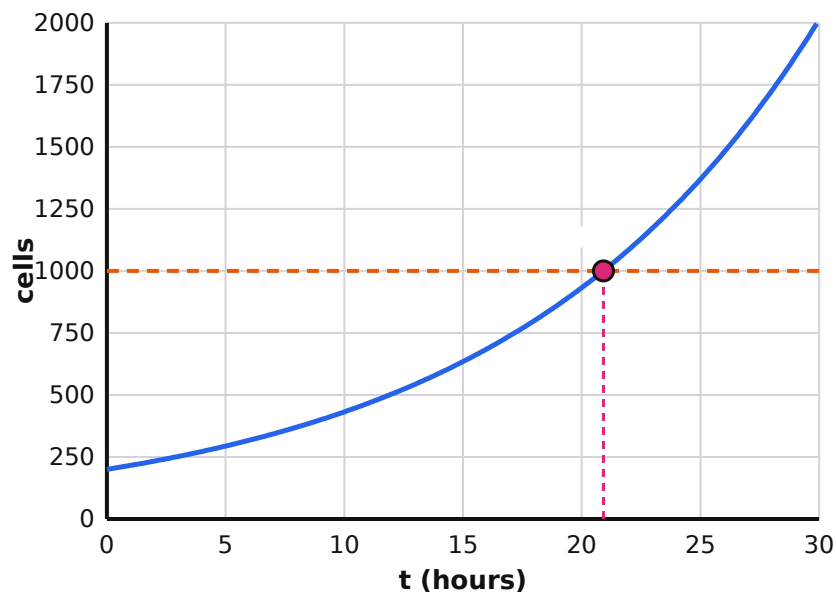
$1.035^{12}=e^{12\ln 1.035}$, so $k=\ln 1.035 \approx \mathbf{0.0344}$, giving $P(t)=12000e^{0.0344t}$.

Monthly factor: $1.035^{1/12} \approx 1.00287$, so the rate is about **0.287%** per month.

Problem 18: Bacteria growth

$200(1.08)^t=1000$, $1.08^t=5$, $t=\frac{\ln 5}{\ln 1.08} \approx \frac{1.6094}{0.0770} \approx \mathbf{20.91}$ hours.

Doubling: $1.08^t=2$, $t=\frac{\ln 2}{\ln 1.08} \approx \mathbf{9.01}$ hours.



The intersection at $t \approx 20.91$ hours.

Problem 19: Compound interest

$3000(1.01125)^{4t} = 5000$, so $1.01125^{4t} = \frac{5}{3}$. Then $4t = \frac{\ln(5/3)}{\ln 1.01125} \approx \frac{0.5108}{0.01119} \approx 45.66$, so $t \approx 11.42$ **years**.

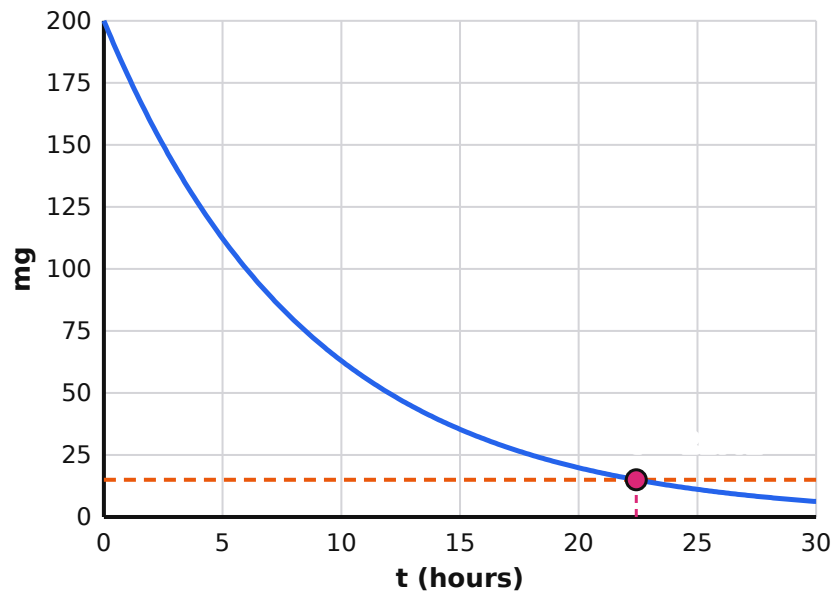
Problem 20: Continuous compounding

Double: $e^{0.032t} = 2$, $t = \frac{\ln 2}{0.032} \approx 21.66$ **years**.

\\$4,000: $e^{0.032t} = \frac{8}{3}$, $t = \frac{\ln(8/3)}{0.032} \approx \frac{0.9808}{0.032} \approx 30.65$ **years**.

Problem 21: Medicine half-life

$\left(\frac{1}{2}\right)^{t/6} = \frac{15}{200} = 0.075$. Then $\frac{t}{6} = \frac{\ln 0.075}{\ln 0.5} \approx 3.737$, so $t \approx 22.42$ **hours**.



The solution $t \approx 22.42$ hours.

Problem 22: Earthquake comparison

- The difference is $7.6 - 5.2 = 2.4$, so the ratio is $10^{2.4} \approx \mathbf{251}$ times.
- $M = 5.2 + \log 250 \approx 5.2 + 2.398 = \mathbf{7.6}$.

Problem 23: pH and decibels

- $-\log[H^+] = 3.8$, so $[H^+] = 10^{-3.8} \approx 1.58 \times 10^{-4}$ mol/L.
- $\log \frac{I}{I_0} = 9.5$, so $\frac{I}{I_0} = 10^{9.5} \approx 3.16 \times 10^9$.