

Algebra 1 · Transformations, Absolute Value and Piecewise Functions · Worksheet Answer Key

Common Core F-BF.B.3, F-IF.C.7b · mathovore.fr

Every solution shows the rule used and the check. Compare your graphs with the figures, paying attention to vertices, open circles and closed circles, and rewrite any problem where your answer differs.

Algebra 1 Common Core: F-BF.B.3 · F-IF.C.7b

Problem 1: Shift down a parabola

$g(x) = f(x) - 6$, so every output decreases by 6. The graph shifts **down 6 units**.

The vertex of f is $(0,0)$, so the vertex of g is $(0,-6)$.

Problem 2: Shift up an absolute value

$g(x) = f(x) + 7$, so every output increases by 7. The graph shifts **up 7 units**.

The vertex moves from $(0,0)$ to $(0,7)$.

Problem 3: Equation from a shifted parabola

The vertex moved from $(0,0)$ to $(5,0)$, which is 5 units right. A move right by 5 uses $x-5$ inside.

$g(x) = f(x-5)$, so $g(x) = (x-5)^2$.

Check with $(6,1)$: $(6-5)^2 = 1$. Check with $(3,4)$: $(3-5)^2 = 4$.

Problem 4: Shift left

$g(x) = f(x+3)$, so the graph shifts **left 3 units**. The vertex is $(-3,0)$.

The y-intercept: $g(0) = |0+3| = 3$. The y-intercept is $(0,3)$.

Problem 5: Add outside or inside

$f(3)+2 = 3^2+2 = 9+2 = 11$.

$f(3+2) = f(5) = 5^2 = 25$.

The results differ because the first adds 2 to the *output* (a vertical shift) and the second adds 2 to the *input* (a horizontal shift). **11 and 25**

Problem 6: Stretch table

Substitute each value:

x	-2	-1	0	1	2
$f(x)$	2	1	0	1	2
$g(x)$	6	3	0	3	6

Each y-value of g is 3 times the matching y-value of f . The graph is a **vertical stretch by a factor of 3**; the vertex stays at $(0,0)$.

Problem 7: Equation from an absolute value graph

The vertex is $(2,-3)$, so $h=2$ and $k=-3$.

$$g(x) = |x-2|-3$$

Check with $(5,0)$: $|5-2|-3 = 3-3 = 0$. Check with $(0,-1)$: $|0-2|-3 = 2-3 = -1$.

Problem 8: Reflect and shrink

Compare $(2,4)$ on f with $(2,-2)$ on g : $-2 = -\frac{1}{2} \cdot 4$. Compare $f(4)=16$ with $g(4)=-8$: $-8 = -\frac{1}{2} \cdot 16$.

So $g(x) = -\frac{1}{2}f(x)$, that is $g(x) = -\frac{1}{2}x^2$.

The graph is **flipped over the x-axis** (the negative sign) and **shrunk vertically by a factor of $\frac{1}{2}$** .

Problem 9: Transform a linear function

(a) $g(x) = (2x+1)+3 = 2x+4$.

(b) $h(x) = 2(x+3)+1 = 2x+6+1 = 2x+7$.

(c) $k(x) = -(2x+1) = -2x-1$.

Notice that shifting the line left 3 units gave the same line as shifting it up 6 units, because the slope is 2. **(a) $2x+4$; (b) $2x+7$; (c) $-2x-1$**

Problem 10: Vertex and range

Here $h=4$ and $k=2$. The vertex is $(4,2)$.

The axis of symmetry is the vertical line through the vertex: $x=4$.

The graph opens up, so the minimum value is **2**.

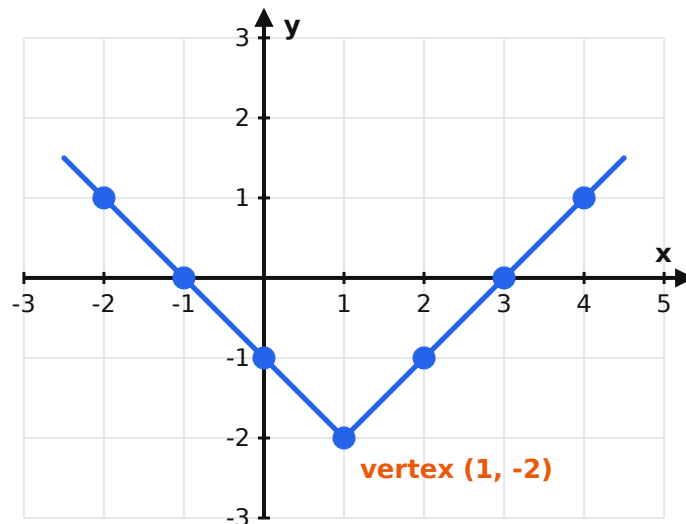
Domain: all real numbers. Range: $y \geq 2$.

Problem 11: Graph an absolute value function

Substitute each x :

x	-2	-1	0	1	2	3	4
y	1	0	-1	-2	-1	0	1

For example, $x=-2$: $|-2-1|-2=3-2=1$. Plot the points and draw two rays meeting at the vertex $(1,-2)$.



The graph of $y=|x-1|-2$ with the seven table points.

Problem 12: Intercepts of an absolute value graph

y-intercept: $g(0)=2|0+1|-4=2-4=-2$, so $(0,-2)$.

x-intercepts: set $2|x+1|-4=0$. Then $2|x+1|=4$ and $|x+1|=2$. So $x+1=2$ or $x+1=-2$, giving $x=1$ or $x=-3$.

x-intercepts $(1,0)$ and $(-3,0)$

Problem 13: An absolute value that opens down

The vertex is $(3,5)$. Because $a = -1 < 0$, the graph opens down and the maximum value is **5**.

y-intercept: $h(0) = -|0-3|+5 = -3+5 = 2$, so $(0,2)$.

x-intercepts: $-|x-3|+5 = 0$ gives $|x-3| = 5$, so $x-3 = 5$ or $x-3 = -5$, and $x = 8$ or $x = -2$. **$(8,0)$ and $(-2,0)$**

Problem 14: Evaluate a piecewise function

$-3 < 0$, so use $x+4$: $f(-3) = -3+4 = 1$.

$-1 < 0$: $f(-1) = -1+4 = 3$.

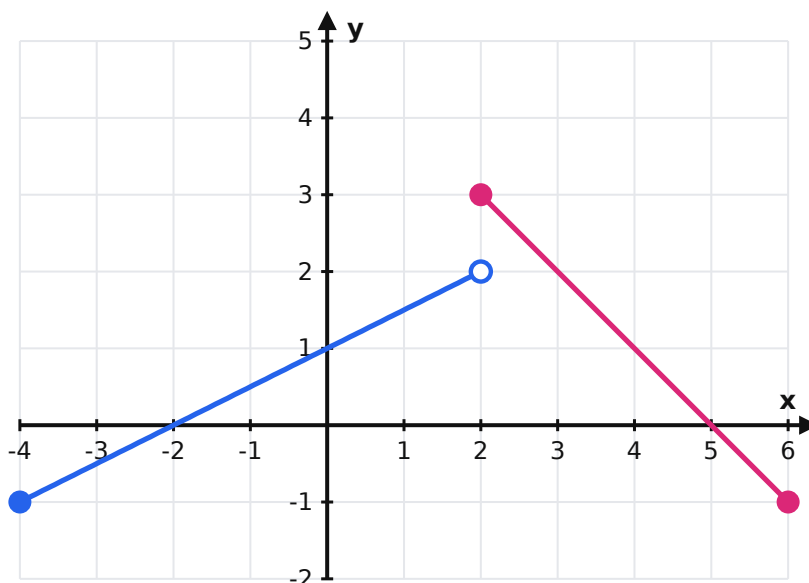
$0 \geq 0$, so use x^2 : $f(0) = 0^2 = 0$.

$2 \geq 0$: $f(2) = 2^2 = 4$. **1, 3, 0, 4**

Problem 15: Graph a piecewise function

Left piece, $x < 2$: $f(-4) = -1$, $f(0) = 1$, and at $x = 2$ the rule gives $\frac{1}{2}(2)+1 = 2$, an **open circle at $(2,2)$** .

Right piece, $x \geq 2$: $f(2) = -2+5 = 3$, a **closed circle at $(2,3)$** ; then $f(3) = 2$ and $f(6) = -1$.



Open circle at $(2,2)$; closed circle at $(2,3)$.

Problem 16: Write a piecewise function

Piece 1: horizontal at $y=2$, with a closed circle at $x=-1$: $f(x)=2$ for $-4 \leq x \leq -1$.

Piece 2: from $(-1,0)$ (open) to $(3,4)$ (closed), slope $\frac{4-0}{3-(-1)}=1$, so $y=x+1$ for $-1 < x \leq 3$.

Piece 3: from $(3,3)$ (open) to $(6,0)$, slope $\frac{0-3}{6-3}=-1$, so $y=6-x$ for $3 < x \leq 6$.

$f(-4)=2$; $f(3)=3+1=4$ (piece 2, because 3 is included there); $f(5)=6-5=1$. **2, 4, 1**

Problem 17: Greatest integer step function

$s(3.7)=3$, because 3 is the greatest integer not exceeding 3.7.

$s(-1.2)=-2$, because $-2 \leq -1.2$ but $-1 > -1.2$.

$s(5)=5$ and $s(-4)=-4$, since integers map to themselves.

$s(0.99)=0$. **3, -2, 5, -4, 0**

Problem 18: Check a point

$g(4)=-2|4-1|+6=-2(3)+6=0$. The y-value is 0, so $(4,0)$ **is on the graph**.

$g(-2)=-2|-2-1|+6=-2(3)+6=0$, not 2. So $(-2,2)$ **is not on the graph** (the point $(-2,0)$ is).

Problem 19: Ride-share booking fee

Adding 4 to every output shifts the graph **up 4 units**. $g(m)=2m+4$.

$g(7)=2(7)+4=14+4=18$. **The 7-mile ride costs \$18.**

Problem 20: Starting to save later

Everything happens 3 weeks later, so replace w with $w-3$: $g(w)=f(w-3)=15(w-3)$ for $w \geq 3$. The graph shifts **right 3 units**.

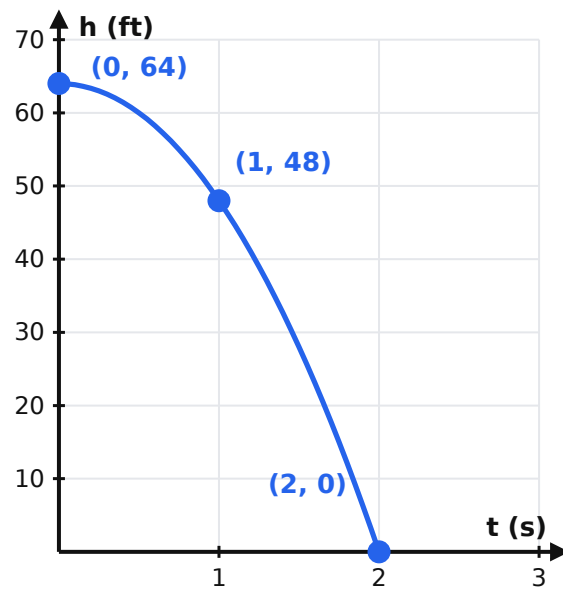
$g(8)=15(8-3)=15(5)=75$. \$75

Problem 21: Dropped ball

$h(t)=-16f(t)+64$: stretch vertically by a factor of 16, flip over the t-axis, then shift up 64 units.

Starting height: $h(0)=64$ feet.

Landing: $-16t^2 + 64 = 0$, so $t^2 = 4$ and $t = 2$ (time cannot be negative). **The ball lands after 2 seconds.**



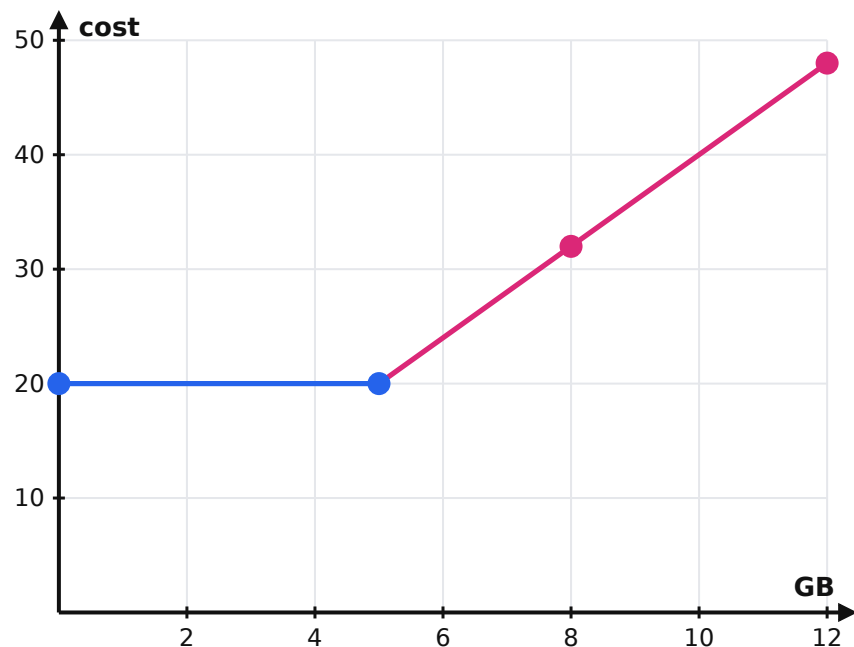
The ball's height $h(t) = -16t^2 + 64$ for $0 \leq t \leq 2$.

Problem 22: Data plan

$C(g) = 20$ for $0 \leq g \leq 5$, and $C(g) = 20 + 4(g - 5)$ for $g > 5$.

$C(3) = 20$. $C(8) = 20 + 4(3) = 32$. $C(12) = 20 + 4(7) = 48$.

\$20, \$32 and \$48



Flat at \$20 up to 5 GB, then \$4 more for each extra GB.

Problem 23: Parking garage

(a) 2.3 hours is in the interval from 2 to 3, where the height is 15. \$15

(b) At $x=3$ the closed circle is at height 15. \$15

(c) Just past 3, the cost jumps to the next step. \$20

Check: $5 \times 3 = 15$ for 2.3 hours (3 started hours), and $5 \times 4 = 20$ for 3.01 hours.

Problem 24: Bottle tolerance

15.9 oz: $E = |15.9 - 16| = |-0.1| = 0.1 \leq 0.2$. **Acceptable.**

16.3 oz: $E = |16.3 - 16| = 0.3 > 0.2$. **Not acceptable.**

$|x - 16| \leq 0.2$ means $-0.2 \leq x - 16 \leq 0.2$, so $15.8 \leq x \leq 16.2$.