

Algebra 1 · Transformations, Absolute Value and Piecewise Functions · Lesson

Common Core F-BF.B.3, F-IF.C.7b · mathovore.fr

A transformation changes a parent function's graph in a predictable way. Adding k outside, $f(x)+k$, shifts the graph up or down. Replacing x with $x+k$ shifts it left or right. Multiplying by k , $k \cdot f(x)$, stretches, shrinks or flips it. Absolute value, piecewise and step functions combine these ideas.

Algebra 1 Common Core: F-BF.B.3 · F-IF.C.7b

Key facts:

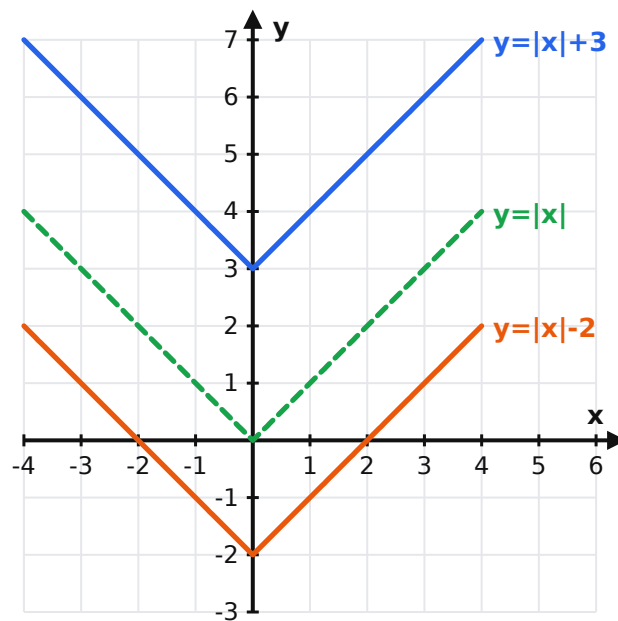
- Standards F-BF.B.3 and F-IF.C.7b: describe how $f(x)+k$, $f(x+k)$ and $kf(x)$ change a graph, and graph absolute value, piecewise-defined and step functions.
- Outside changes act on y and are intuitive; inside changes act on x and work backward: $x - 4$ moves right 4.
- The graph of $a|x - h| + k$ has its vertex at (h, k) and opens up when $a > 0$ and down when $a < 0$.
- On a piecewise graph, a closed circle includes the endpoint and an open circle excludes it.
- A step function has horizontal pieces and jumps, like a price charged per box or per hour or part of an hour.

What is a transformation of a function?

A **transformation** changes the graph of a parent function in a predictable way. Algebra 1 uses three parent functions: linear $f(x) = x$, quadratic $f(x) = x^2$ and absolute value $f(x) = |x|$. Each change to the equation causes one matching change in the graph.

Transformation rules (k is a number)

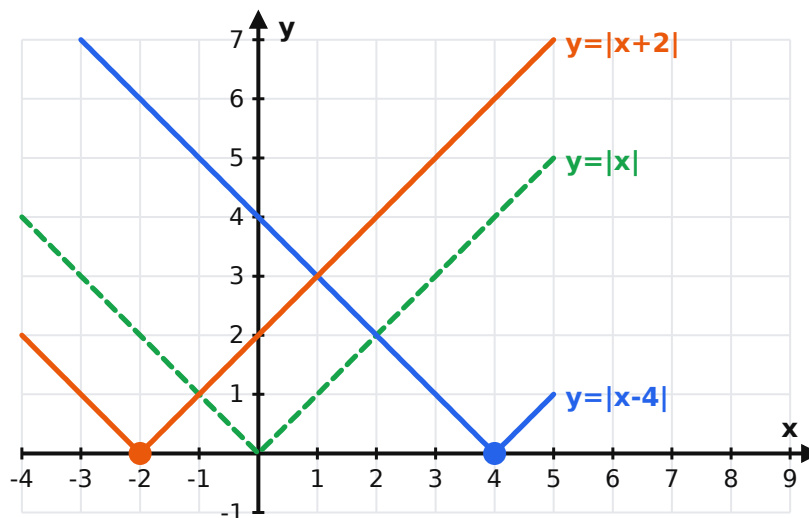
New function	Effect on the graph
$f(x)+k$	Vertical shift: up k units if $k > 0$, down $ k $ units if $k < 0$
$f(x+k)$	Horizontal shift: left k units if $k > 0$, right $ k $ units if $k < 0$
$k \cdot f(x)$	Vertical stretch if $ k > 1$, shrink if $0 < k < 1$, flip over the x-axis if $k < 0$



Adding a number outside moves the whole V up or down: the vertex goes to (0, 3), (0, 0) and (0, -2).

How does $f(x + k)$ move a graph?

A number added inside the parentheses changes the input before the function acts, so the shift is horizontal and works backward. For $f(x) = |x|$, the function $f(x-4) = |x-4|$ equals 0 when $x=4$, so its vertex moves to (4,0): 4 units right. Likewise $|x+2|$ equals 0 at $x=-2$, so it moves 2 units left.

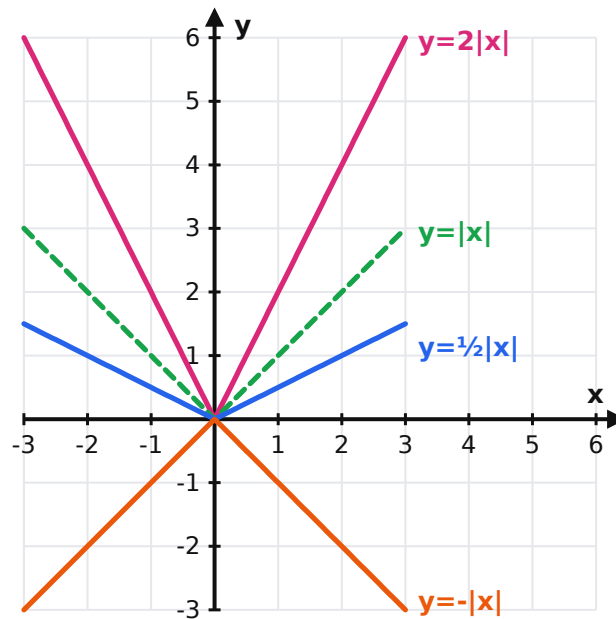


Inside changes work backward: $|x-4|$ moves 4 units right and $|x+2|$ moves 2 units left.

What does k times $f(x)$ do?

Multiplying the output by k scales every y-value. For $f(x) = |x|$, the point (1,1) becomes (1,2) on

$2|x|$, $(2,1)$ on $\frac{1}{2}|x|$ and $(1,-1)$ on $-|x|$. Points on the x-axis never move, because $k \cdot 0 = 0$.



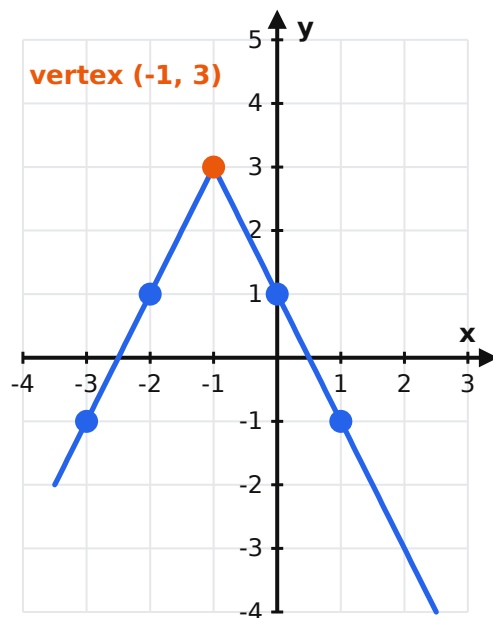
Multiplying the output by 2 stretches the V, by $\frac{1}{2}$ shrinks it, and by -1 flips it over the x-axis.

How do you graph an absolute value function?

Write the function as $g(x) = a|x-h|+k$. The **vertex** is (h,k) . The graph opens up if $a > 0$ and down if $a < 0$, and its two branches have slopes a and $-a$.

1. Plot the vertex (h,k) .
2. Move 1 unit right and a units up (down if a is negative) to plot a second point.
3. Mirror that point across the vertical line $x=h$.
4. Draw two rays from the vertex through these points.

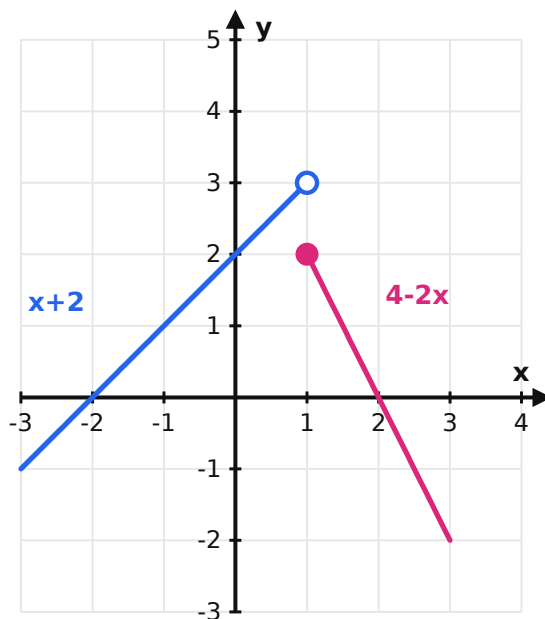
Worked example. Graph $g(x) = -2|x+1|+3$. Here $h = -1$, $k = 3$ and $a = -2$. The vertex is $(-1,3)$. One unit right gives $(0,1)$, and its mirror is $(-2,1)$.



The graph of $g(x) = -2|x+1|+3$: vertex $(-1, 3)$, opening down, with slopes -2 and 2 .

What are piecewise and step functions?

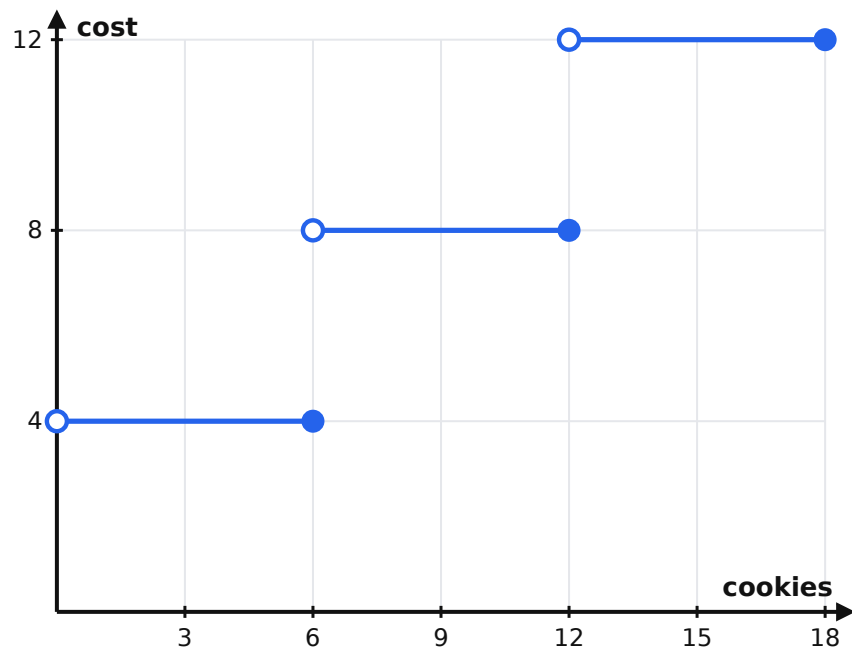
A **piecewise function** uses a different rule on each part of its domain. For example, $f(x) = x+2$ when $x < 1$, and $f(x) = 4-2x$ when $x \geq 1$. At the boundary, a **closed circle** marks a point that belongs to the graph and an **open circle** marks a point that does not.



Open circle at $(1, 3)$: that point is not included. Closed circle at $(1, 2)$: that point is included.

A **step function** is a piecewise function whose pieces are horizontal segments. It models

prices that jump, such as \$4 for each box of up to 6 cookies. The cost of 7 cookies is \$8, not \$4.67.



Cookies sold in boxes of 6 at \$4 per box: the cost jumps at 6, 12 and 18, forming stairs.

Transformations, Absolute Value and Piecewise Functions: frequently asked questions

Why does $f(x - 3)$ move the graph to the right?

The output $f(x - 3)$ reaches a given value when the input is 3 larger than before. The vertex of $(x - 3)^2$ occurs when $x - 3 = 0$, which is $x = 3$, so every point shifts 3 units right.

What is the difference between $f(x) + k$ and $f(x + k)$?

$f(x) + k$ adds k to every output and shifts the graph vertically. $f(x + k)$ adds k to every input and shifts the graph horizontally in the opposite direction. For $f(x) = x^2$ and $k = 3$, the vertex moves to $(0, 3)$ in the first case and to $(-3, 0)$ in the second.

How do you find the vertex of an absolute value function?

Write the function as $a|x - h| + k$. The vertex is the point (h, k) . For $y = 3|x + 5| - 2$, $h = -5$ and $k = -2$, so the vertex is $(-5, -2)$.

What does an open circle mean on a piecewise graph?

An open circle means the graph approaches that point but does not include it. The value at that x -coordinate is given by another piece, usually shown with a closed circle.

How is a step function different from a linear function?

A linear function changes steadily and has a graph that is one connected line. A step function stays constant on an interval and then jumps, so its graph looks like stairs and has gaps between pieces.