

Algebra 1 · Graphing Linear Inequalities and Systems of Inequalities · Worksheet Answer Key

Common Core A-REI.D.12, A-CED.A.3 · mathovore.fr

Each solution gives the boundary line, the line type, the test point and the shading decision, so you can see where a graph went wrong. Compare the described region with your own drawing, and confirm any point you chose by substituting it into every inequality.

Algebra 1 Common Core: A-REI.D.12 · A-CED.A.3

Problem 1: Test a point

Substitute $x=2$ and $y=1$: $1 < 3(2)$, which is $1 < 6$. True.

Yes, $(2,1)$ is a solution.

Problem 2: Points on the boundary

- a. $2(3)+3(2)=12$, and $12 \geq 12$ is true because $\geq$ allows equality. **Yes.**
- b. $2(1)+3(2)=8$, and $8 \geq 12$ is false. **No.**

Problem 3: Solid or dashed

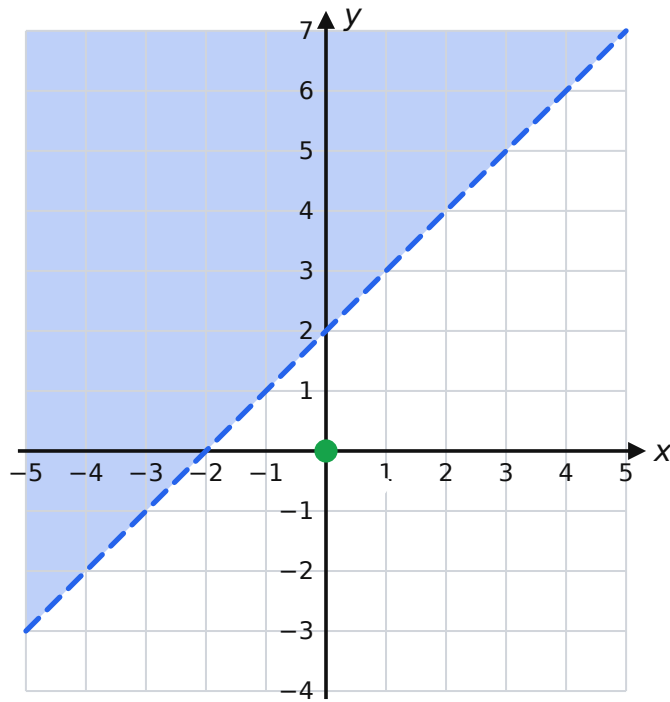
- a. $>$: **dashed.**
- b. $\leq$: **solid.**
- c. $<$: **dashed.**
- d. $\geq$: **solid.**

A line is solid only when the symbol includes equality.

Problem 4: Graph $y > x + 2$

The boundary $y = x + 2$ has y-intercept 2 and slope 1. The symbol $>$ makes it dashed.

Test $(0,0)$: $0 > 0 + 2$ is false, so shade the side that does *not* contain the origin: **above the line.**

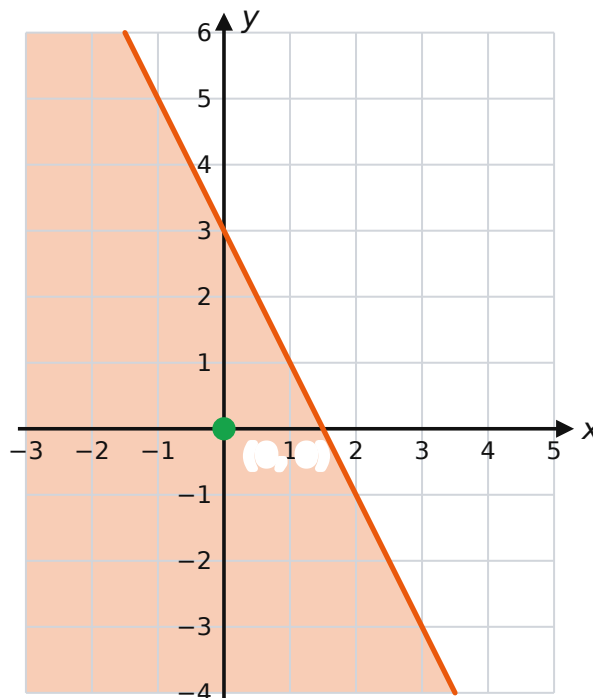


The shaded region is above the dashed line.

Problem 5: Graph $y \leq -2x + 3$

The boundary $y = -2x + 3$ passes through $(0, 3)$ and $(1, 1)$. The symbol $\leq$ makes it solid.

Test $(0, 0)$: $0 \leq 3$ is true, so shade the side with the origin: **below the line**.

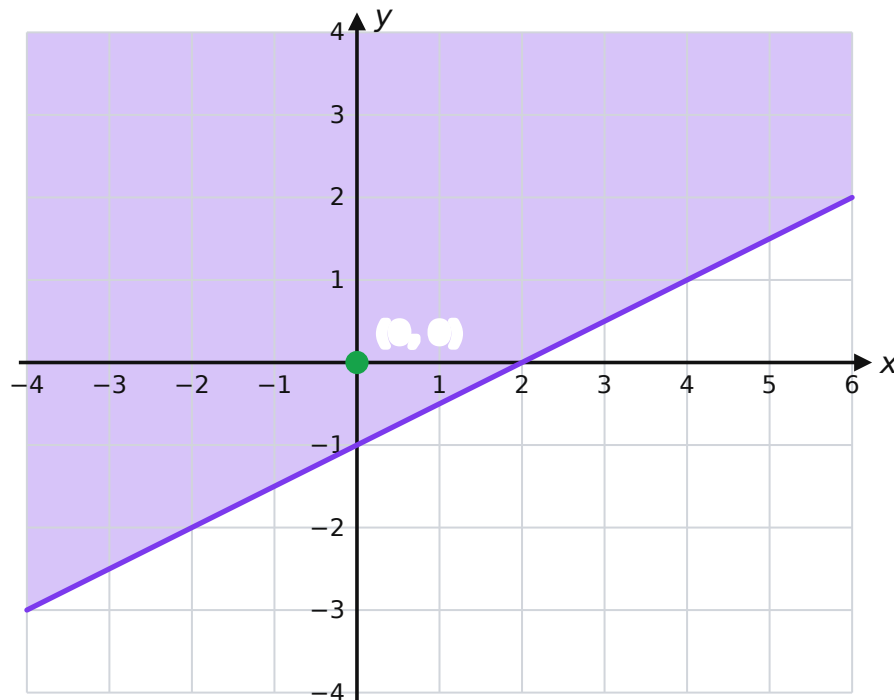


Solid line, shaded below.

Problem 6: Graph with a fractional slope

The boundary passes through $(0, -1)$ and, using rise 1 and run 2, through $(2, 0)$ and $(4, 1)$. It is solid.

Test $(0, 0)$: $0 \geq -1$ is true, so shade **above the line**, where the origin is.

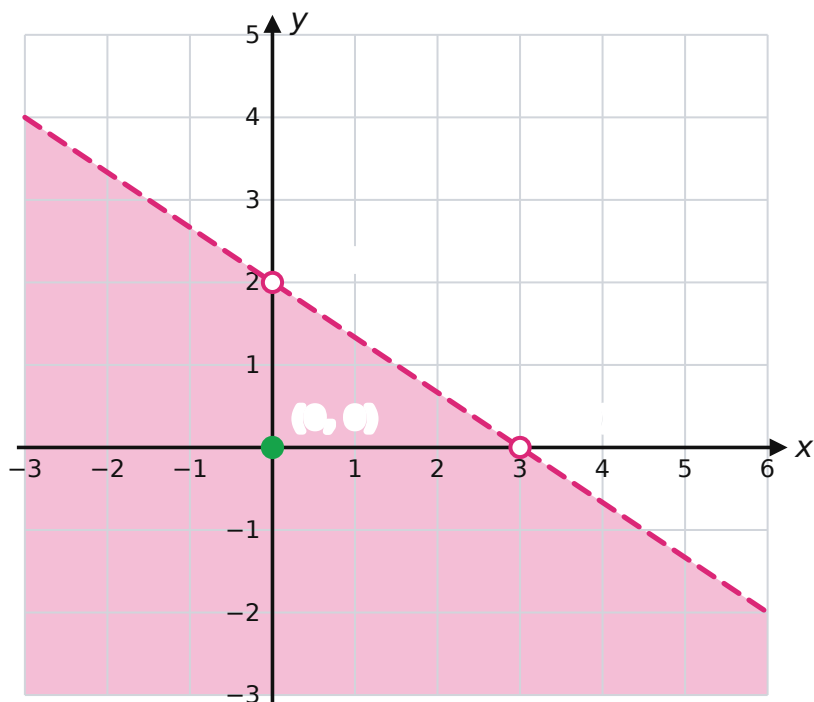


Solid line, shaded above.

Problem 7: Standard form

Set $y = 0$: $2x = 6$, so $x = 3$. Set $x = 0$: $3y = 6$, so $y = 2$. The boundary joins $(3, 0)$ and $(0, 2)$ and is dashed.

Test $(0, 0)$: $0 < 6$ is true, so shade the side that contains the origin.



Dashed boundary with the origin side shaded.

Problem 8: Divide by a negative

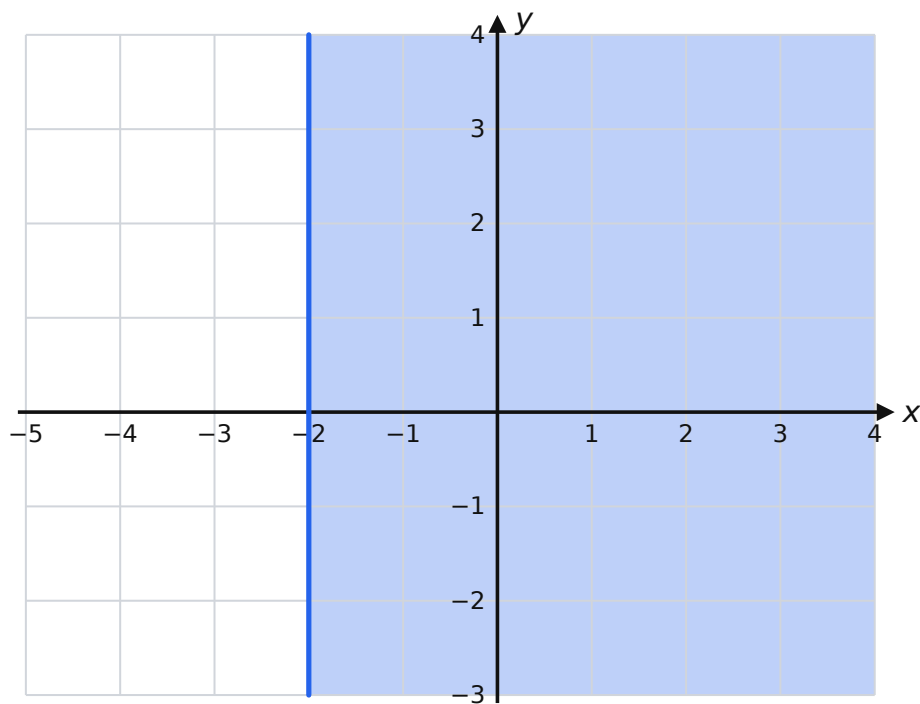
Divide every term by -3 and reverse the symbol: $y \leq -2x + 3$.

The symbol $\leq$ gives a **solid** line. The test point $(0,0)$ gives $0 \leq 3$, true, so the shading is **below** the line.

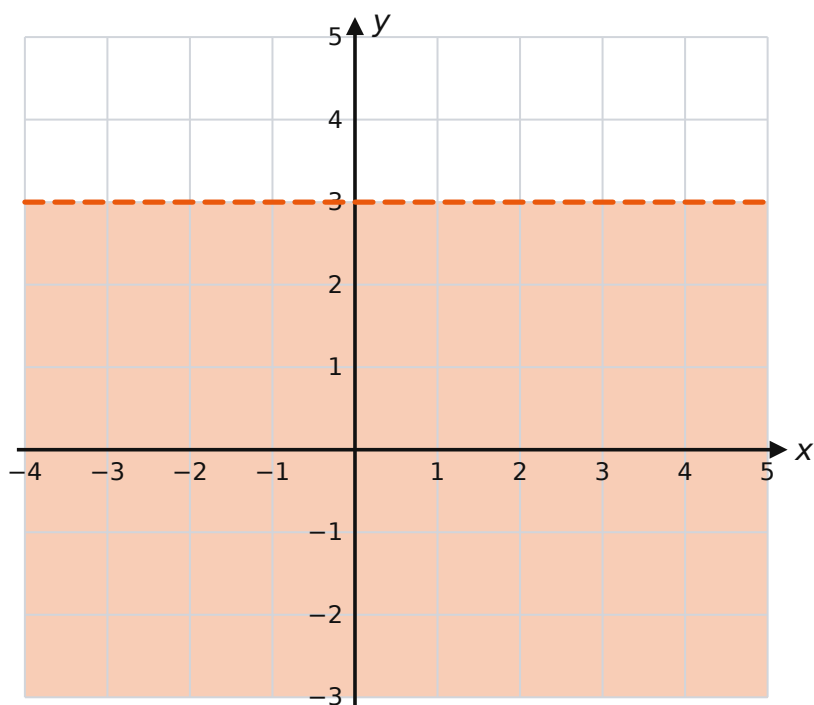
Problem 9: Vertical and horizontal

$x \geq -2$: draw a solid vertical line through $x = -2$ and shade everything to its right.

$y < 3$: draw a dashed horizontal line through $y = 3$ and shade everything below it.



The graph of $x \geq -2$.



The graph of $y < 3$.

Problem 10: Write from a graph: dashed

The line passes through $(0,1)$ and $(2,5)$, so the slope is $\frac{5-1}{2-0} = 2$ and the equation is $y = 2x + 1$.

The line is dashed, so the symbol is strict. The shaded side is below the line, and the origin is shaded: $0 < 2(0) + 1$ is true.

The inequality is $y < 2x + 1$.

Problem 11: Write from a graph: solid

The line passes through $(0, 2)$ and $(2, 0)$, so the slope is $\frac{0-2}{2-0} = -1$ and the equation is $y = -x + 2$.

The line is solid and the shading is above it. Check a shaded point such as $(2, 2)$: $2 \geq -2 + 2 = 0$ is true.

The inequality is $y \geq -x + 2$.

Problem 12: Check points in a system

- $(0, 0)$: $0 > -1$ true; $0 \leq 5$ true. Solution.
- $(3, 1)$: $1 > 2$ false. Not a solution.
- $(2, 2)$: $2 > 1$ true; $2 \leq 3$ true. Solution.
- $(4, 2)$: $2 > 3$ false. Not a solution.
- $(-1, 4)$: $4 > -2$ true; $4 \leq 6$ true. Solution.

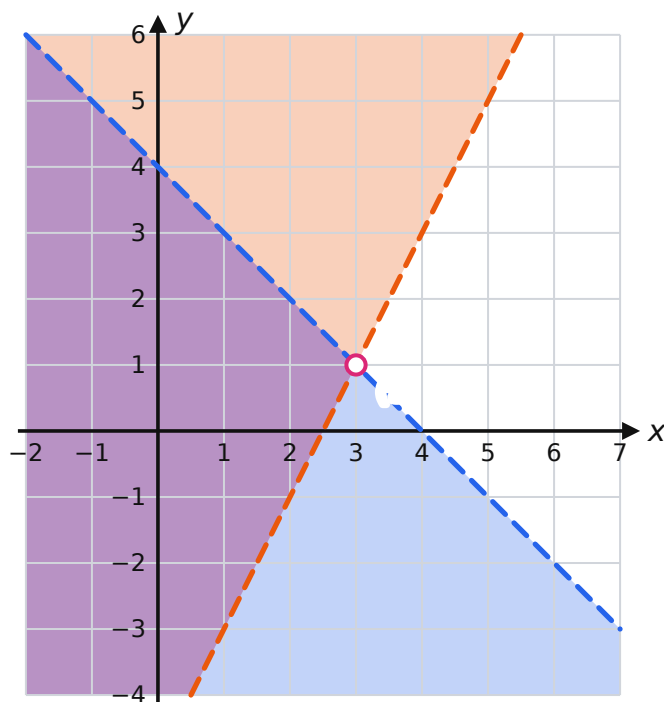
$(0, 0)$, $(2, 2)$ and $(-1, 4)$ are solutions.

Problem 13: Graph a system

Both boundaries are dashed. $y = -x + 4$ passes through $(0, 4)$ and $(4, 0)$; shade below. $y = 2x - 5$ passes through $(0, -5)$ and $(3, 1)$; shade above.

Set $-x + 4 = 2x - 5$: $9 = 3x$, so $x = 3$ and $y = 1$. The boundaries meet at $(3, 1)$.

Test the corner: $1 < -3 + 4 = 1$ is false, so $(3, 1)$ is not a solution. The solution region is the overlap to the left of that point.



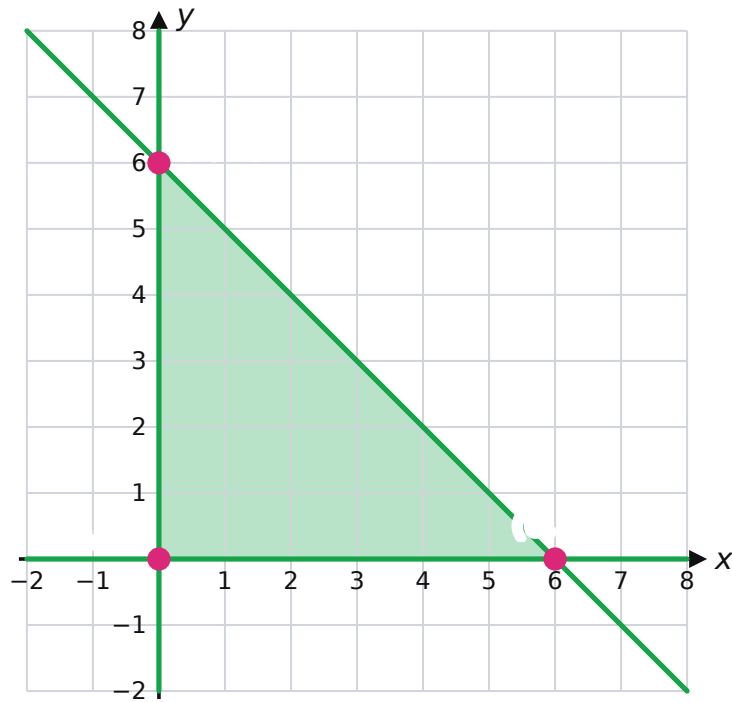
The overlap is the solution region.

Problem 14: A triangular region

$x \geq 0$ shades right of the y-axis and $y \geq 0$ shades above the x-axis. The line $x+y=6$ joins $(6,0)$ and $(0,6)$; shade below it. All three lines are solid.

The corners are $(0,0)$, $(6,0)$ **and** $(0,6)$.

Test $(2,3)$: $2 \geq 0$, $3 \geq 0$ and $2+3=5 \leq 6$ are all true, so it is a solution.



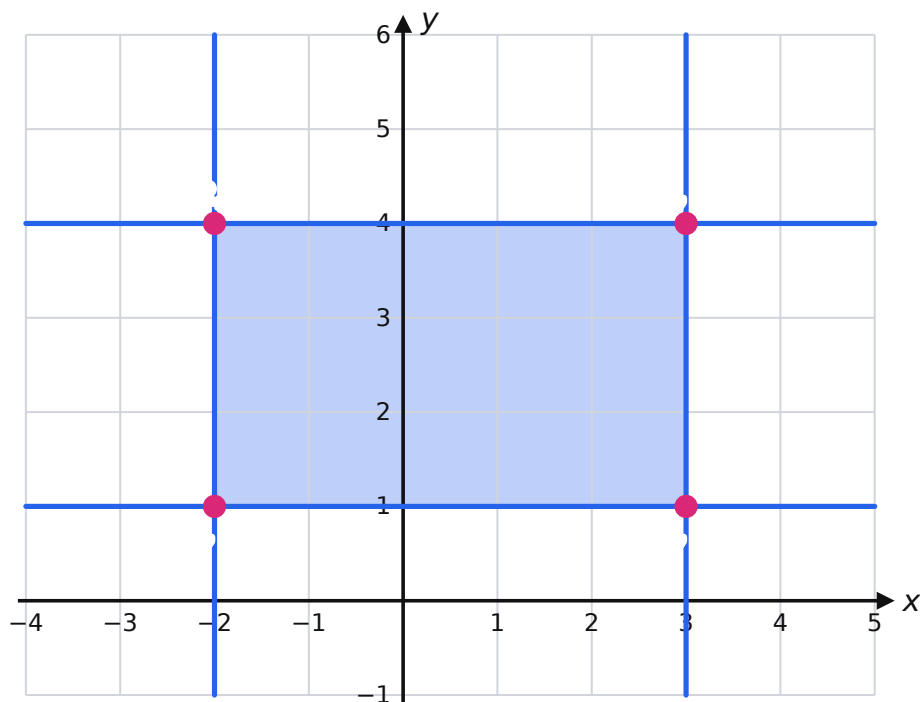
The feasible triangle.

Problem 15: A rectangular region

The inequalities form four solid boundaries: $x = -2$, $x = 3$, $y = 1$ and $y = 4$.

The corners are $(-2,1)$, $(3,1)$, $(3,4)$ **and** $(-2,4)$.

The width is $3 - (-2) = 5$ and the height is $4 - 1 = 3$, so the area is $5 \times 3 = \mathbf{15 \text{ square units}}$.



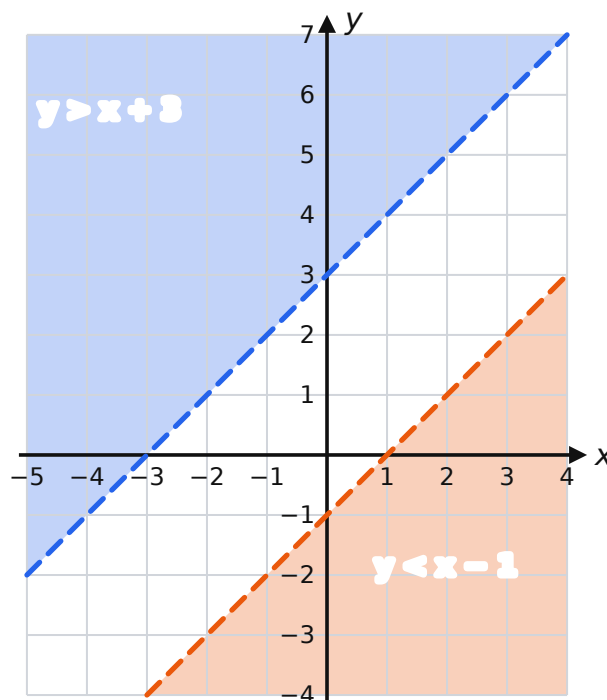
The rectangular solution region.

Problem 16: No overlap

The boundary lines have the same slope 1, so they are parallel. $y > x + 3$ shades above the upper line, and $y < x - 1$ shades below the lower line.

A solution would need $x + 3 < y < x - 1$, which would make $x + 3 < x - 1$, or $3 < -1$. That is false.

The shaded regions never overlap, so there is no solution.



The two shaded regions are separated.

Problem 17: Concert budget

The total cost must be at most \$150: $15x + 25y \leq 150$.

- $15(4) + 25(3) = 60 + 75 = 135 \leq 150$. Yes.
- $15(5) + 25(3) = 75 + 75 = 150 \leq 150$. Yes, exactly the budget.
- $15(2) + 25(5) = 30 + 125 = 155 > 150$. No.

Problem 18: Bake sale limits

At most 60 items: $x + y \leq 60$. At least 20 cookies: $y \geq 20$. Counts cannot be negative: $x \geq 0$.

- $(35, 25)$: $60 \leq 60$ and $25 \geq 20$. Possible.
- $(45, 20)$: $45 + 20 = 65 > 60$. Not possible.
- $(30, 15)$: $15 < 20$. Not possible.
- $(20, 20)$: $40 \leq 60$ and $20 \geq 20$. Possible.

Possible: $(35, 25)$ and $(20, 20)$.

Problem 19: Yard work hours

Time: $2x + y \leq 10$. Counts: $x \geq 0$ and $y \geq 0$.

The line $2x + y = 10$ meets the axes at $(5, 0)$ and $(0, 10)$. The corners are $(0, 0)$, $(5, 0)$ **and** $(0, 10)$.

3 lawns and 4 walks: $2(3)+4=10 \leq 10$. Fits.

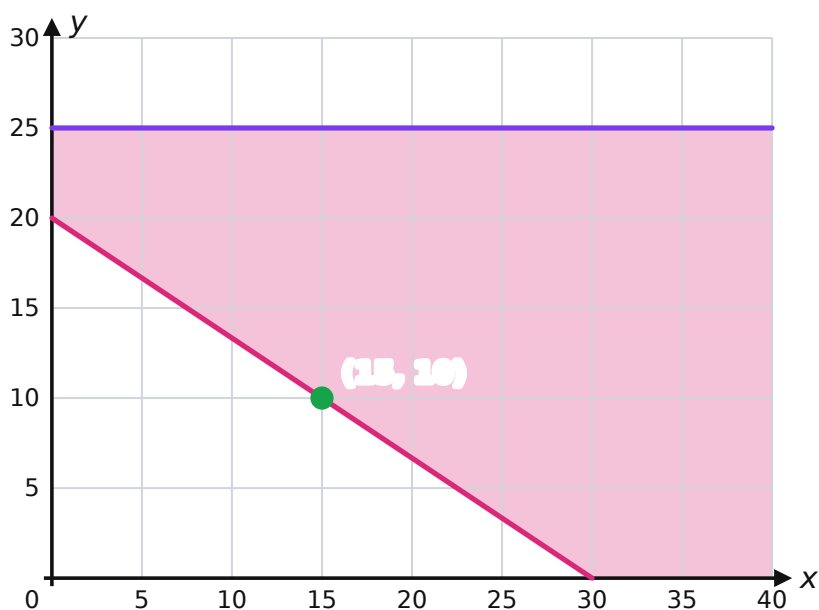
4 lawns and 3 walks: $2(4)+3=11 > 10$. Does not fit.

Problem 20: Lemonade goal

Earnings: $2x+3y \geq 60$. Cookie limit: $y \leq 25$.

- $(15,10)$: $30+30=60 \geq 60$ and $10 \leq 25$. Meets both.
- $(10,10)$: $20+30=50 < 60$. Fails.
- $(30,25)$: $60+75=135 \geq 60$ and $25 \leq 25$. Meets both.

(15, 10) and (30, 25) work.



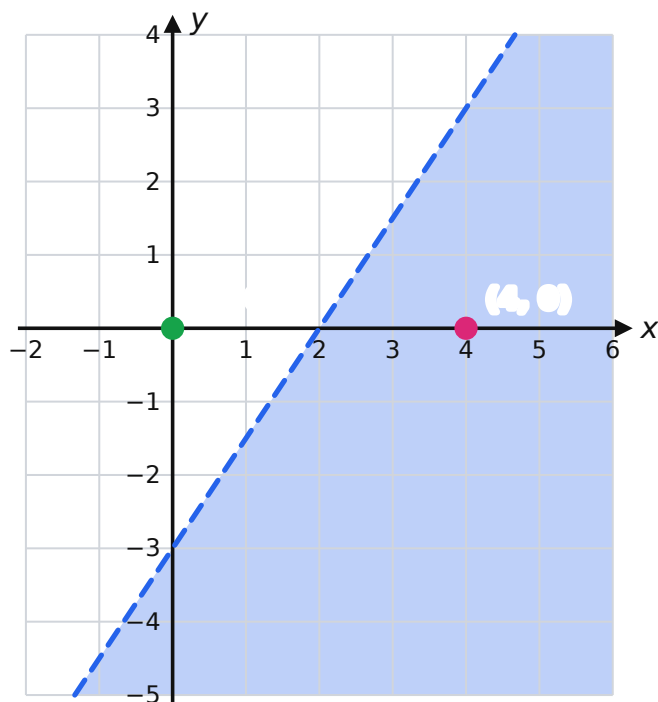
The shaded region meets both conditions.

Problem 21: Find the error

Mistake 1: the symbol $>$ is strict, so the line must be dashed.

Mistake 2: test $(0,0)$: $3(0)-2(0)=0 > 6$ is false, so the origin side must *not* be shaded.

Boundary: $y=0$ gives $x=2$; $x=0$ gives $y=-3$. Test $(4,0)$: $12 > 6$ is true, so shade the side containing $(4,0)$, which is below and to the right of the line.



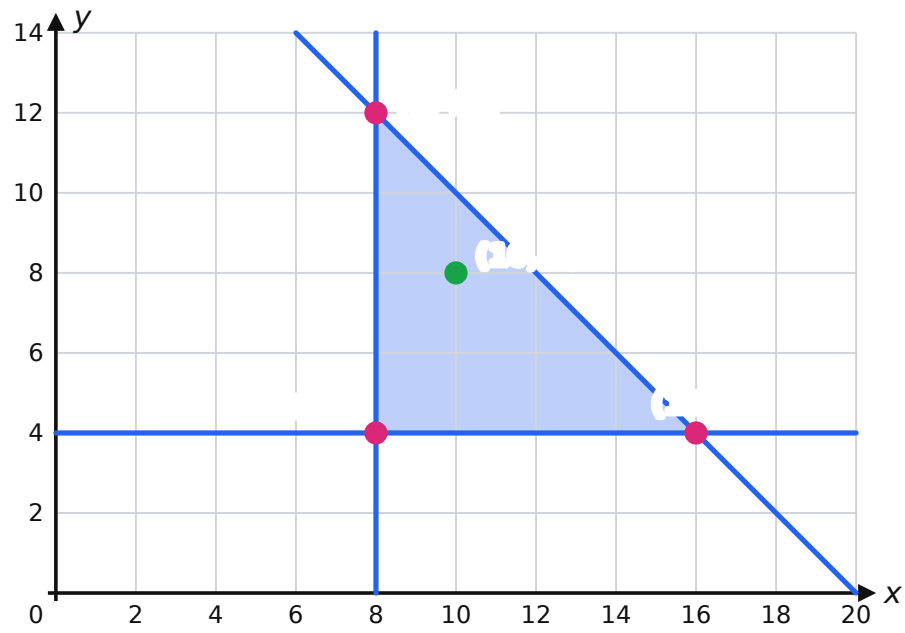
The correct graph.

Problem 22: Garden fence

The perimeter is $2x+2y \leq 40$, which simplifies to $x+y \leq 20$. With the minimums: $x \geq 8$ and $y \geq 4$.

Corners: $x=8$, $y=4$ gives $(8,4)$. With $y=4$, $x+4=20$ gives $(16,4)$. With $x=8$, $8+y=20$ gives $(8,12)$.

Test $(10,8)$: $10 \geq 8$, $8 \geq 4$ and $18 \leq 20$ are all true, so **a 10 ft by 8 ft garden is allowed.**



The feasible region has corners $(8, 4)$, $(16, 4)$ and $(8, 12)$.