

Algebra 1 · Graphing Linear Inequalities and Systems of Inequalities · Lesson

Common Core A-REI.D.12, A-CED.A.3 · mathovore.fr

A linear inequality in two variables, such as $y > 2x - 1$, has a half-plane of solutions bounded by a line. Draw the boundary dashed for $<$ and $>$ or solid for $\leq$ and $\geq$, then shade the side that makes a test point true. A system's solution is the region where all the shadings overlap.

Algebra 1 Common Core: A-REI.D.12 · A-CED.A.3

Key facts:

- Standards A-REI.D.12 and A-CED.A.3: graph linear inequalities and systems, and model constraints.
- Solid line for $\leq$ and $\geq$; dashed line for $<$ and $>$.
- Use a test point not on the line, such as $(0,0)$, to decide which side to shade.
- Multiplying or dividing by a negative number reverses the inequality symbol.
- The solution set of a system is the overlap of the half-planes; a feasible region lists all allowed combinations.

What is a linear inequality in two variables?

A **linear inequality in two variables**, such as $y > 2x - 1$ or $3x + 2y \leq 12$, replaces the equals sign of a line with $<$, $\leq$, $>$ or $\geq$. Its solutions are ordered pairs (x,y) that make it true. Those points fill a whole region of the plane called a **half-plane**, bounded by the line you get when you change the inequality symbol to $=$.

Vocabulary

Boundary line: the line that separates solutions from non-solutions.

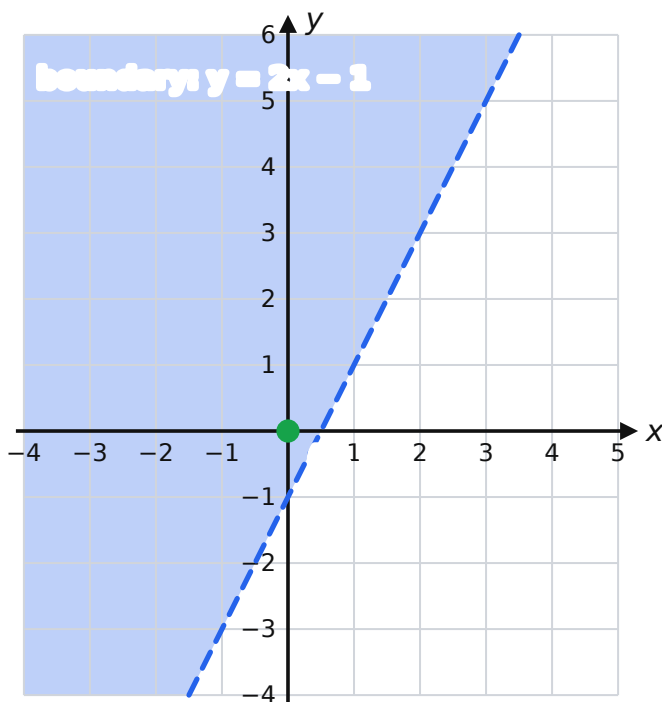
Solid line: used for $\leq$ and $\geq$; points on the line are solutions.

Dashed line: used for $<$ and $>$; points on the line are not solutions.

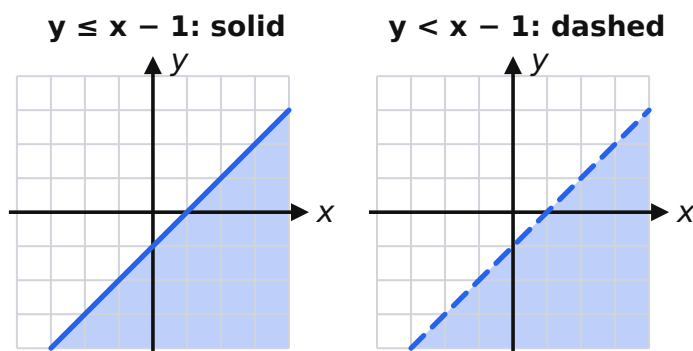
How do you graph a linear inequality?

1. Graph the boundary line, replacing the inequality symbol with an equals sign.
2. Make the line dashed for $<$ or $>$, and solid for $\leq$ or $\geq$.
3. Pick a test point that is not on the line. The origin $(0,0)$ is easiest.
4. Substitute the test point. If the result is true, shade the side containing it. If false, shade the other side.

Worked example. Graph $y > 2x - 1$. The boundary $y = 2x - 1$ is dashed. Test $(0,0)$: $0 > -1$ is true, so shade the side that contains the origin, which is above the line.

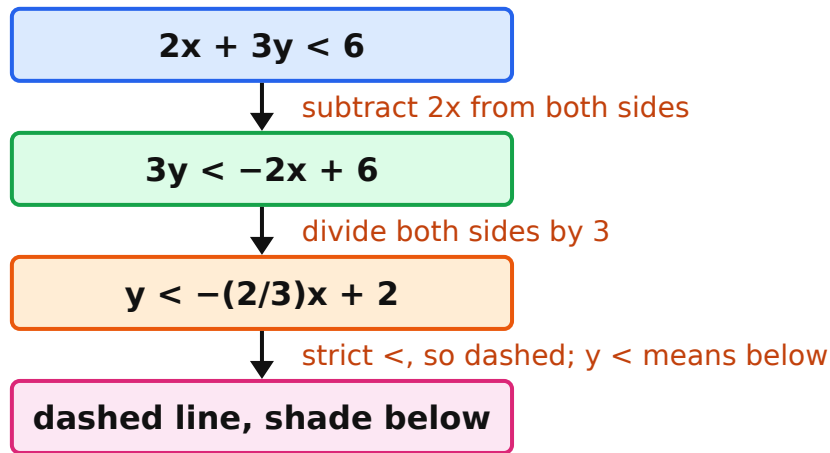


The dashed line $y = 2x - 1$ with the half-plane above it shaded. The test point $(0,0)$ is a solution.



A solid boundary includes its points; a dashed boundary leaves them out.

When a line is written as $Ax + By < C$, solve for y or test a point. If you divide or multiply by a negative number, reverse the inequality symbol: $-2y > 8$ becomes $y < -4$.

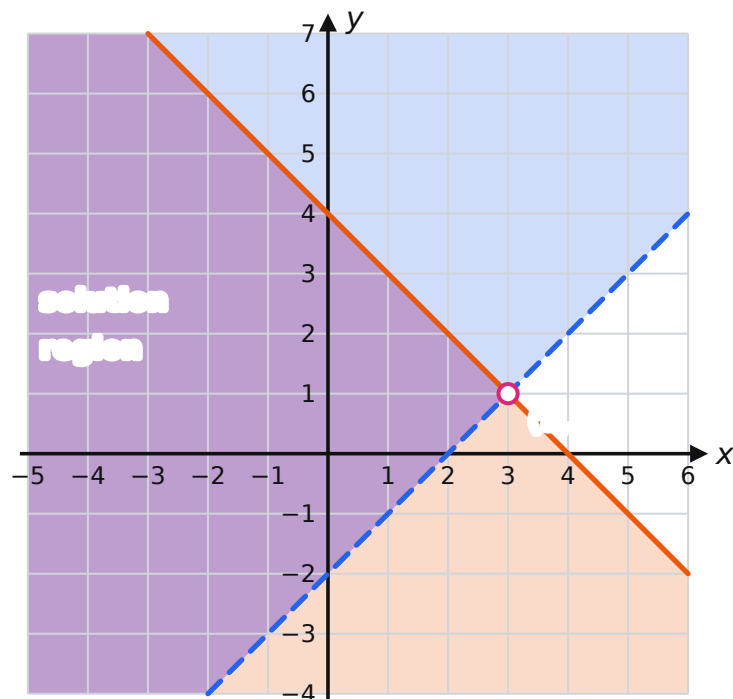


Rewriting a standard-form inequality so that the shading direction is clear.

How do you graph a system of inequalities?

Graph each inequality on the same grid. The **solution set** of the system is the region where all the shadings overlap. Check by testing one point inside the overlap in every inequality.

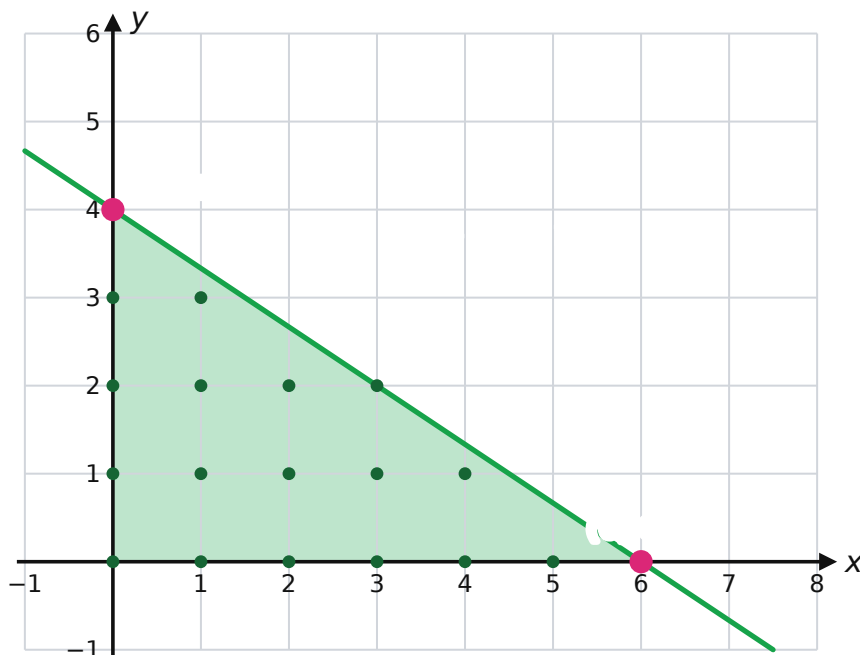
Worked example. Graph $y > x - 2$ and $y \leq -x + 4$. The first line is dashed and shaded above; the second is solid and shaded below. The lines meet at $(3,1)$. That corner is an open circle because $1 > 1$ is false. Test $(0,0)$: $0 > -2$ and $0 \leq 4$ are both true, so $(0,0)$ is in the solution region.



The darker overlap is the solution set of the system.

How do constraints model real situations?

A **constraint** is a limit written as an inequality. Priya buys x cupcakes at \$2 and y muffins at \$3 with at most \$12. The constraints are $2x+3y \leq 12$, $x \geq 0$ and $y \geq 0$. The overlap is a triangle called the **feasible region**. Only whole-number points make sense for items you can count.



Each dot is a possible purchase. For example, $(3, 2)$ costs $3(2) + 2(3) = \$12$.

Graphing Linear Inequalities and Systems of Inequalities: frequently asked questions

How do I know whether to use a solid or a dashed line?

Look at the symbol. Use a solid line for $\leq$ and $\geq$, because points on the line make the statement true. Use a dashed line for $<$ and $>$, because points on the line do not.

Which side of the line should I shade?

Pick a test point that is not on the line, usually $(0, 0)$, and substitute it. If the inequality is true, shade the side with that point. If it is false, shade the opposite side.

Why do I flip the inequality symbol when I divide by a negative number?

Multiplying by a negative reverses the order of numbers on the number line. For example, $2 < 5$ but $-2 > -5$. The same reversal keeps $-3y > 6$ true when it becomes $y < -2$.

What is the solution set of a system of inequalities?

It is the set of all points that make every inequality true, which is the region where the shaded half-planes overlap. If the half-planes do not overlap, the system has no solution.

What is a feasible region?

A feasible region is the part of the graph where every constraint in a real-world problem is satisfied. Each point in it is a combination that follows all the limits, such as a budget and a minimum amount.