

Algebra 1 · Systems of Linear Equations · Lesson

Common Core A-REI.C.5, A-REI.C.6, A-REI.D.11, A-CED.A.3 · mathovore.fr

A system of linear equations is a set of two or more linear equations with the same variables. Its solution is the ordered pair that makes every equation true, which is the point where the graphs cross. You can find it by graphing, by substitution, or by elimination, and then check it in both equations.

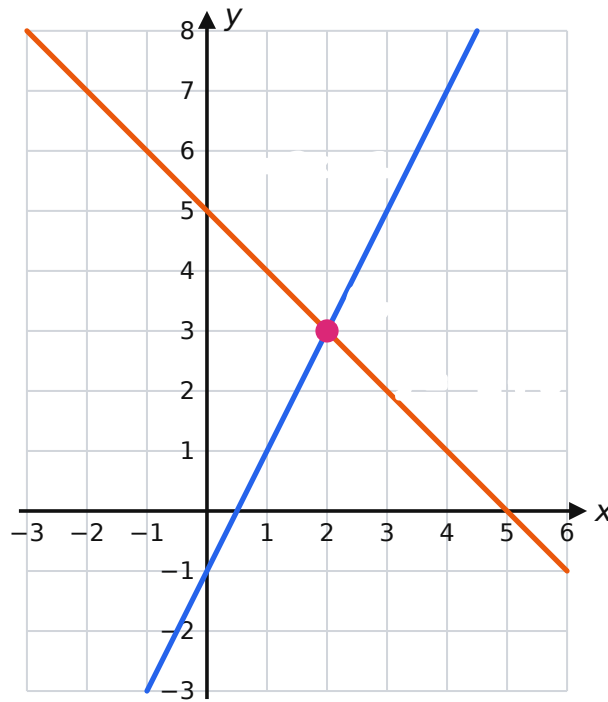
Algebra 1 Common Core: A-REI.C.5 · A-REI.C.6 · A-REI.D.11 · A-CED.A.3

Key facts:

- Standards A-REI.C.5, A-REI.C.6, A-REI.D.11 and A-CED.A.3: solve systems by graphing, substitution and elimination, and model real situations.
- Two lines meet once, never, or everywhere: one solution, no solution, or infinitely many.
- Substitution works best when a variable is already isolated; elimination works best when coefficients match or are opposites.
- Adding equations is valid because equal quantities added to equal quantities stay equal.
- The x-coordinates where two graphs meet solve $f(x) = g(x)$ for lines, parabolas and exponentials alike.

What is a system of linear equations?

A **system of linear equations** is two or more linear equations that use the same variables. A **solution** is an ordered pair (x, y) that makes every equation true. On a graph it is the point where the lines cross. More generally, the x-coordinates where two graphs meet solve $f(x) = g(x)$, whether the graphs are lines, parabolas or exponential curves.



The lines $y = 2x - 1$ and $y = -x + 5$ cross at $(2, 3)$, the only solution.

Vocabulary

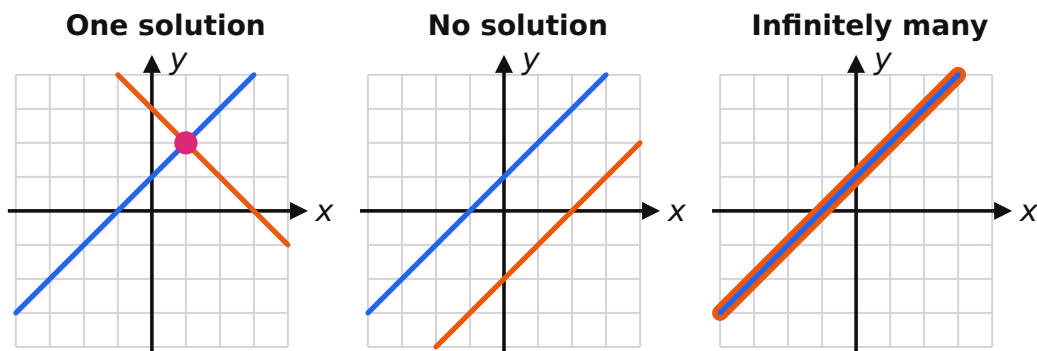
Substitution: replace one variable with an equal expression taken from the other equation.

Elimination: add or subtract the equations so that one variable cancels.

Consistent system: a system with at least one solution. A system with none is **inconsistent**.

How many solutions can a system have?

Two lines can cross once, never meet, or lie on top of each other. Different slopes give one solution. Equal slopes with different y-intercepts give none. Identical lines give infinitely many solutions.

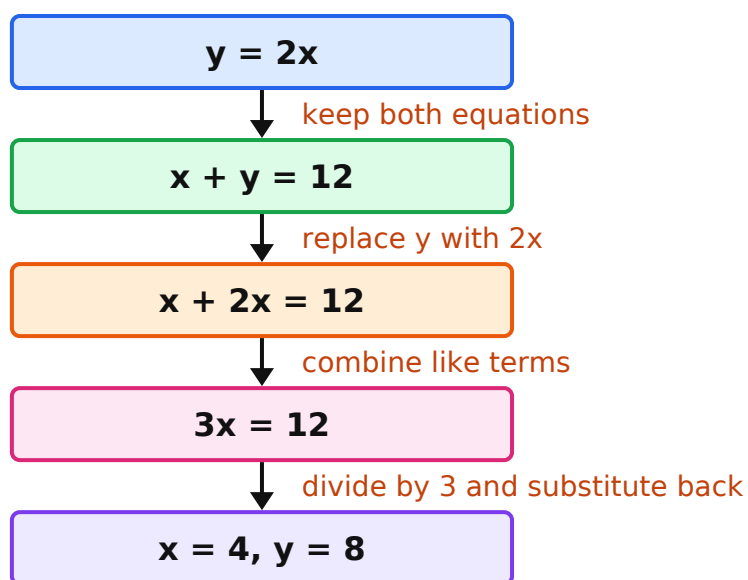


The three possible pictures for a system of two linear equations.

How do you solve by substitution?

1. Solve one equation for a variable, or use one that is already solved.
2. Substitute that expression into the other equation.
3. Solve the resulting one-variable equation.
4. Substitute back to find the other variable.
5. Check the pair in both original equations.

Worked example. Solve $y = 2x$ and $x + y = 12$. Substituting gives $x + 2x = 12$, so $x = 4$ and $y = 8$. Check: $4 + 8 = 12$ and $8 = 2(4)$.



Substitution turns two equations in two variables into one equation in one variable.

How do you solve by elimination, and why does it work?

If $A = B$ and $C = D$, then $A + C = B + D$. So any pair that satisfies both equations also satisfies their sum, and the sum can be built to contain only one variable. When no variable cancels, first multiply an equation by a nonzero number. That replaces the equation with one

that has exactly the same solutions.

Worked example. Solve $3x+2y=16$ and $5x-2y=16$. The y -terms are opposites, so add:
 $8x=32$ and $x=4$. Then $3(4)+2y=16$ gives $y=2$.

$$\begin{array}{rclcrcl}
 3x & + & 2y & = & 16 \\
 5x & - & 2y & = & 16 \\
 \hline
 8x & & 0y & = & 32
 \end{array}$$

y-terms cancel:
 $2y + (-2y) = 0$

Line up the like terms, add, and one variable disappears.

How do you solve a real-world system?

Define two variables, write one equation for the count and one for the money, then solve. A drama club sold 90 tickets at \$7 (adult) and \$4 (child) for \$510 in all. Let a be adult tickets and c child tickets: $a+c=90$ and $7a+4c=510$. Substituting $c=90-a$ gives $3a+360=510$, so $a=50$ and $c=40$.

Ticket	Price	Number	Sales
Adult	\$7	50	\$350
Child	\$4	40	\$160
Total		90	\$510

Check the context: $50+40=90$ tickets and $\$350+\$160=\$510$.

Always check a pair in both original equations. A pair that satisfies only one equation is not a solution of the system.

Systems of Linear Equations: frequently asked questions

What does the solution of a system of equations mean?

The solution is an ordered pair that makes every equation in the system true at the same time. On a graph, it is the point where the lines intersect. For $y=x+1$ and $y=-x+5$, the solution is $(2,3)$.

When should I use substitution instead of elimination?

Use substitution when one equation already has a variable alone, such as $y=3x$, or when a

coefficient is 1. Use elimination when both equations are in the form $Ax + By = C$ and some coefficients match or are opposites.

Why does adding two equations keep the solution?

Adding equal quantities to equal quantities gives equal quantities. If a pair makes both equations true, it makes their sum true as well. Multiplying one equation by a nonzero number also keeps the same solutions.

How can I tell that a system has no solution or infinitely many?

Write both equations as $y = mx + b$. The same slope with different intercepts means parallel lines and no solution. The same slope and the same intercept mean one line written twice, so every point on it is a solution.

How do I solve $f(x) = g(x)$ when the graphs are not lines?

Graph both functions, or make a table of values, and find where they meet. The x-coordinate of each intersection point is a solution of the equation. For $y = x^2$ and $y = x + 6$, the graphs meet at $x = -2$ and $x = 3$.