

Algebra 1 · Arithmetic and Geometric Sequences · Lesson

Common Core F-IF.A.3, F-BF.A.2, F-LE.A.2 · mathovore.fr

A sequence is a function whose inputs are the positive integers. An arithmetic sequence adds a common difference d each term, so $a_n = a_1 + (n - 1)d$. A geometric sequence multiplies by a common ratio r , so $a_n = a_1 r^{(n-1)}$. Each can also be written recursively from the previous term.

Algebra 1 Common Core: F-IF.A.3 · F-BF.A.2 · F-LE.A.2

Key facts:

- Standards F-IF.A.3, F-BF.A.2 and F-LE.A.2: sequences as functions, explicit and recursive formulas, and modeling with arithmetic and geometric sequences.
- Arithmetic: $a_n = a_1 + (n-1)d$; recursive $a_n = a_{n-1} + d$.
- Geometric: $a_n = a_1 \cdot r^{n-1}$; recursive $a_n = r \cdot a_{n-1}$.
- Arithmetic sequences grow linearly; geometric sequences grow or decay exponentially.
- To identify the type, test consecutive differences first, then consecutive ratios.

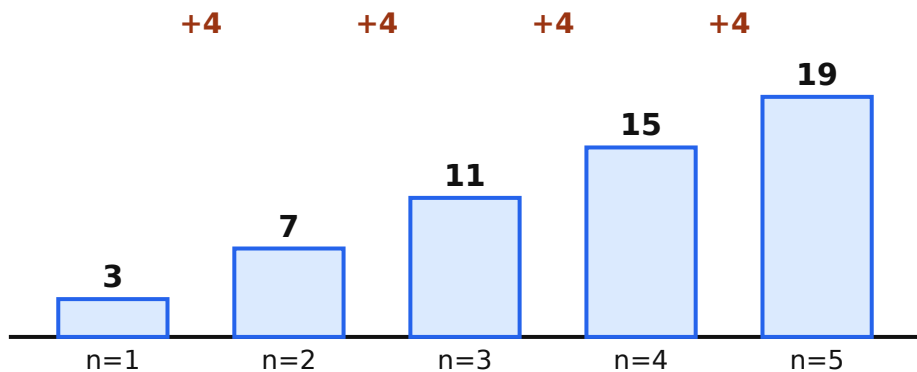
What is a sequence?

A **sequence** is a function whose inputs are the positive integers 1, 2, 3, and so on. The input n is the position and the output a_n is the **term**. The first term is a_1 , the second is a_2 , and a_n is the n th term. Because the inputs are whole numbers, the graph is a set of separate points, not a connected line.

What is an arithmetic sequence?

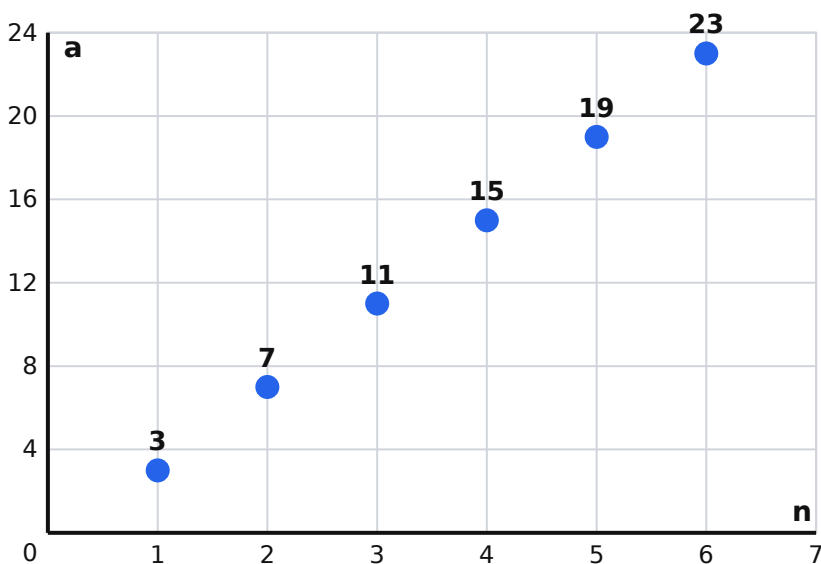
Arithmetic sequence

An **arithmetic sequence** adds the same number each time. That number is the **common difference** d .



An arithmetic sequence: each term is the previous term plus the common difference $d = 4$.

The sequence 3, 7, 11, 15, 19 has $d = 4$. Its **recursive formula** gives each term from the one before: $a_1 = 3$ and $a_n = a_{n-1} + 4$ for $n \geq 2$. Its **explicit formula** gives any term directly: $a_n = a_1 + (n-1)d = 3 + 4(n-1)$, so $a_{20} = 3 + 4(19) = 79$. The graph is a line, so an arithmetic sequence is a linear function.

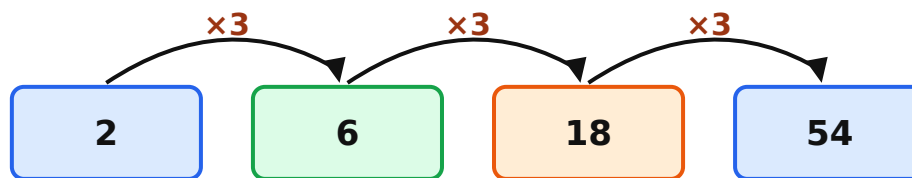


The terms of an arithmetic sequence, plotted as points (n, a_n) , lie on a straight line with slope d .

What is a geometric sequence?

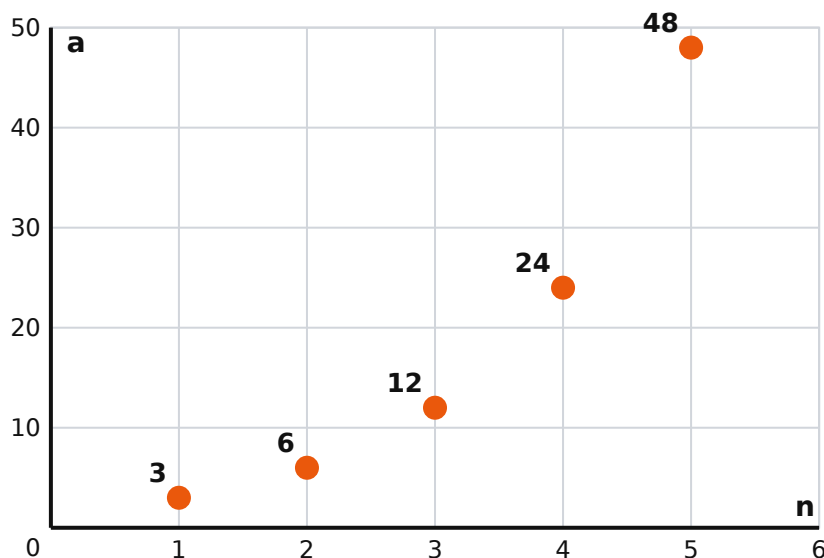
Geometric sequence

A **geometric sequence** multiplies by the same number each time. That number is the **common ratio** r .



A geometric sequence: each term is the previous term times the common ratio $r=3$.

The sequence 2,6,18,54 has $r=3$. Recursive: $a_1=2$ and $a_n=3 \cdot a_{n-1}$. Explicit: $a_n = a_1 \cdot r^{n-1} = 2 \cdot 3^{n-1}$, so $a_6 = 2 \cdot 3^5 = 486$. A geometric sequence is an exponential function of n .



The terms of a geometric sequence curve upward (for $r>1$): the points are not on a line.

How do you tell the two types apart and write a formula?

1. Subtract consecutive terms. If the differences are all equal, it is arithmetic.
2. Divide consecutive terms. If the ratios are all equal, it is geometric.
3. Write the formula with a_1 and d or r .
4. Check one term you already know.

Worked example. For 5,10,20,40 the differences are 5,10,20 (not equal) but every ratio is 2, so it is geometric with $a_n = 5 \cdot 2^{n-1}$. Check: $a_4 = 5 \cdot 8 = 40$.

	Arithmetic	Geometric
Rule	add d	multiply by r
Recursive	$a_n = a_{n-1} + d$	$a_n = r \cdot a_{n-1}$
Explicit	$a_n = a_1 + (n - 1)d$	$a_n = a_1 \cdot r^{n-1}$

How do sequences model real situations?

A savings jar that gets \$6 added each week is arithmetic. A video whose views double daily is geometric. Linear growth adds the same amount each step, while exponential growth multiplies, so a geometric sequence with $r > 1$ eventually passes any arithmetic one.

Arithmetic and Geometric Sequences: frequently asked questions

What is the difference between an arithmetic and a geometric sequence?

An arithmetic sequence adds the same number each time, such as 4,7,10,13. A geometric sequence multiplies by the same number each time, such as 4,8,16,32.

What is the difference between a recursive and an explicit formula?

A recursive formula needs the previous term and a starting value, such as $a_n = a_{n-1} + 4$ with $a_1 = 3$. An explicit formula gives the n th term directly, such as $a_n = 3 + 4(n-1)$.

How do you find the 25th term of an arithmetic sequence?

Use $a_n = a_1 + (n-1)d$. For $a_1 = 7$ and $d = 3$, $a_{25} = 7 + 24(3) = 79$. You do not need to list all 25 terms.

How do you find the common ratio?

Divide any term by the term before it. For 6,18,54, $18 \div 6 = 3$ and $54 \div 18 = 3$, so $r = 3$. A ratio can be a fraction or negative.

Why is a sequence a function?

Each position n has exactly one term a_n . The domain is the positive integers, so the graph is a set of separate points.