

Algebra 1 · Graphing Quadratic Functions · Worksheet Answer Key

Common Core F-IF.C.7a, F-IF.C.8a, F-IF.B.4, A-SSE.B.3b · mathovore.fr

Every solution below shows the formula, the substitution and the final answer in bold.
Compare each step with your own work, and re-check any vertex by substituting its x-coordinate back into the function.

Algebra 1 Common Core: F-IF.C.7a · F-IF.C.8a · F-IF.B.4 · A-SSE.B.3b

Problem 1: Name the coefficients

- a. Compare with ax^2+bx+c : $a=3$, $b=-5$, $c=2$.
- b. The missing x -term means $b=0$: $a=-1$, $b=0$, $c=4$.

Problem 2: Opens up or down

- a. $a=-2<0$, so it opens **down** and the vertex is a **maximum**. The y-intercept is $f(0)=-4$, the point $(0,-4)$.
- b. $a=\frac{1}{2}>0$, so it opens **up** and the vertex is a **minimum**. The y-intercept is $(0,-3)$.

Problem 3: Axis of symmetry

- a. $x=-\frac{8}{2(1)}=-4$. $x=-4$
- b. $x=-\frac{-12}{2(2)}=3$. $x=3$
- c. $x=-\frac{5}{2(-1)}=\frac{5}{2}$. $x=\frac{5}{2}$

Problem 4: Vertex from standard form

- a. $x=-\frac{-6}{2}=3$ and $y=9-18+5=-4$. Vertex: $(3,-4)$.
- b. $x=-\frac{8}{2(-2)}=2$ and $y=-2(4)+16-3=5$. Vertex: $(2,5)$.

Problem 5: Read vertex form

- a. Compare with $a(x-h)^2+k$: $h=3$, $k=1$. Vertex $(3,1)$, axis $x=3$. Since $a=2>0$, the minimum value is **1**.
- b. Write $x+2$ as $x-(-2)$: $h=-2$, $k=5$. Vertex $(-2,5)$, axis $x=-2$. Since $a=-1<0$, the maximum value is **5**.

Problem 6: Zeros from factored form

Set each factor to 0: $x+2=0$ gives $x=-2$, and $x-6=0$ gives $x=6$. The x-intercepts are $(-2,0)$ and $(6,0)$.

The axis is halfway between them: $x = \frac{-2+6}{2} = 2$.

At $x=2$: $y=(4)(-4)=-16$. Vertex: $(2,-16)$.

Problem 7: Complete a table

Substitute each value. For example, $x=-1$: $1+4+1=6$, and $x=2$: $4-8+1=-3$.

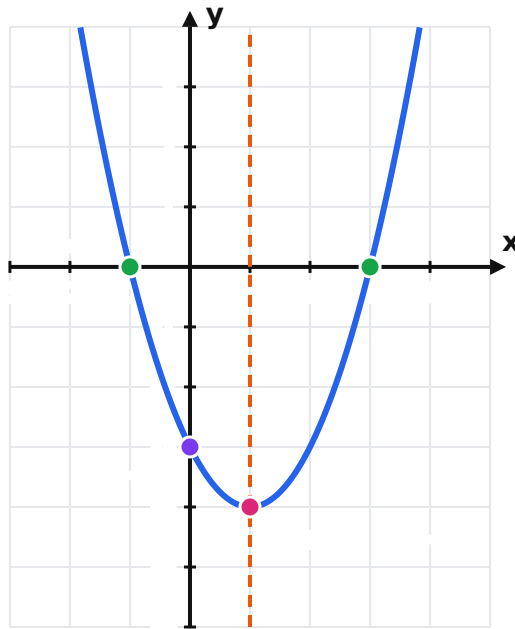
x	-1	0	1	2	3	4	5
y	6	1	-2	-3	-2	1	6

The lowest value is -3 at $x=2$, and the values repeat on both sides of it. Vertex: $(2,-3)$. Axis: $x=2$.

Problem 8: Read a graph

Reading the graph:

- Vertex: $(1,-4)$, the lowest point.
- Axis of symmetry: $x=1$.
- x-intercepts: $(-1,0)$ and $(3,0)$.
- y-intercept: $(0,-3)$.
- The graph never goes below -4 , so the range is $y \geq -4$.



Features of $y = x^2 - 2x - 3$.

Problem 9: Effect of a

Test the point with $x = 2$: $y = x^2$ gives 4, $y = 2x^2$ gives 8, and $y = \frac{1}{2}x^2$ gives 2.

Curve C passes through (2,4), curve B through (1,2) (so $y = 2x^2$), and curve A through (2,2). So **A is $y = \frac{1}{2}x^2$, B is $y = 2x^2$, C is $y = x^2$.**

The graph of $y = -2x^2$ has the same width as B but **opens down**, because a is negative.

Problem 10: Intercepts from standard form

At $x = 0$, $y = 6$: the y-intercept is (0,6).

Set $y = 0$ and factor: $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$, so $x = 2$ or $x = 3$. The x-intercepts are (2,0) and (3,0).

Problem 11: Maximum or minimum value

Since $a = -1 < 0$, the parabola opens down, so there is a **maximum**. The axis is $x = -\frac{6}{2(-1)} = 3$, and $f(3) = -9 + 18 - 5 = 4$. The maximum value is **4**, at $x = 3$.

Zeros: $-(x^2 - 6x + 5) = -(x - 1)(x - 5) = 0$, so $x = 1$ and $x = 5$.

Problem 12: Factored to standard form

Multiply: $x^2 + 5x - 3x - 15 = x^2 + 2x - 15$.

The axis is $x = -\frac{2}{2} = -1$, which is halfway between the zeros 3 and -5 . At $x = -1$: $y = (-4)(4) = -16$. Vertex: $(-1, -16)$.

Problem 13: Complete the square

Half of 8 is 4, and $4^2 = 16$. Add and subtract 16:

$$y = (x^2 + 8x + 16) - 16 + 7 = (x + 4)^2 - 9.$$

Vertex form: $y = (x + 4)^2 - 9$. Vertex: $(-4, -9)$.

Problem 14: No x-intercepts

Half of -10 is -5 , and $(-5)^2 = 25$. So $y = (x^2 - 10x + 25) - 25 + 30 = (x - 5)^2 + 5$.

The vertex is $(5, 5)$, so the minimum value is **5**. The parabola opens up and its lowest point is above the x-axis, so **it never reaches** $y = 0$.

Problem 15: Leading coefficient 2

Factor 2 from the first two terms: $y = 2(x^2 - 6x) + 10$. Add and subtract 9 inside:

$$y = 2(x^2 - 6x + 9) - 18 + 10 = 2(x - 3)^2 - 8.$$

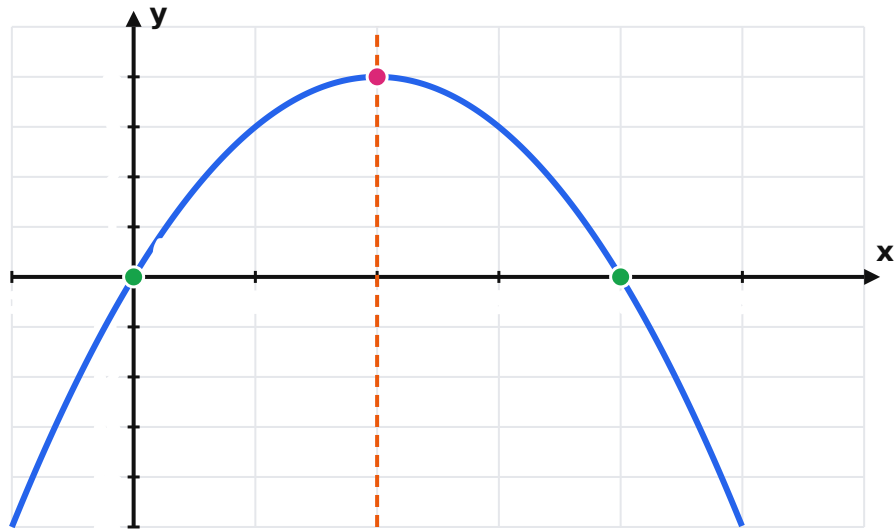
Vertex form: $y = 2(x - 3)^2 - 8$, vertex $(3, -8)$.

Zeros: $2(x - 3)^2 = 8$, so $(x - 3)^2 = 4$ and $x - 3 = \pm 2$. The zeros are $x = 1$ and $x = 5$.

Problem 16: Increasing and decreasing

The vertex is $(2, 4)$, the peak of the graph.

- Increasing: $x < 2$.
- Decreasing: $x > 2$.
- $f(x) > 0$: the graph is above the x-axis between the zeros 0 and 4, so $0 < x < 4$.

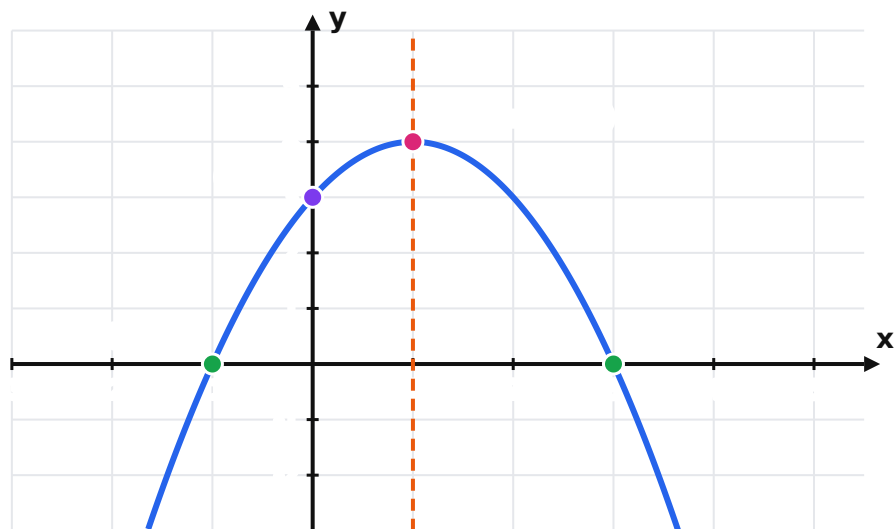


The graph of $f(x) = -x^2 + 4x$, with the vertex and intercepts marked.

Problem 17: Equation from a graph

Start with $y = a(x-1)^2 + 4$. Substitute $(3,0)$: $0 = a(2)^2 + 4$, so $4a = -4$ and $a = -1$.

Vertex form: $y = -(x-1)^2 + 4$. Expand: $-(x^2 - 2x + 1) + 4 = -x^2 + 2x + 3$.



A parabola with vertex $(1,4)$ that passes through $(3,0)$.

Problem 18: Compare two functions

For f : the axis is $x=3$ and $f(3)=9-18+11=2$. The minimum value of f is 2.

For g : the values 5, 2, 1, 2, 5 are symmetric around $x=2$, so the lowest value is $g(2)=1$.

Since $1 < 2$, **function g has the lower minimum.**

Problem 19: Three forms, one parabola

Expand the vertex form: $(x-3)^2 - 4 = x^2 - 6x + 9 - 4 = x^2 - 6x + 5$.

Expand the factored form: $(x-1)(x-5) = x^2 - 5x - x + 5 = x^2 - 6x + 5$.

All three match. **Vertex form** shows the vertex $(3, -4)$; **factored form** shows the zeros 1 and 5; **standard form** shows the y-intercept 5.

Problem 20: Equation from zeros and a point

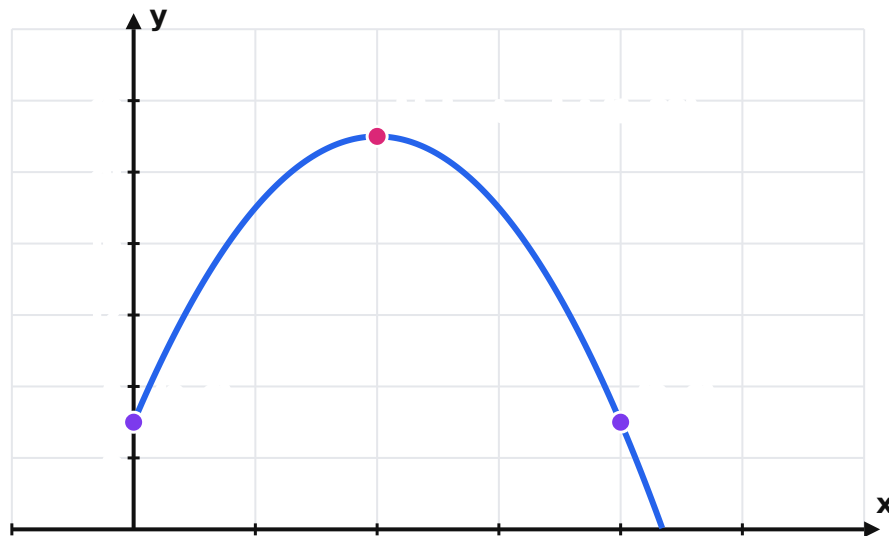
Factored form: $y = a(x-2)(x-8)$. Substitute $(5, -9)$: $-9 = a(3)(-3) = -9a$, so $a = 1$.

Factored: $y = (x-2)(x-8)$. Standard: $y = x^2 - 10x + 16$.

The axis is halfway between 2 and 8, $x = 5$, and $y = -9$ there. Vertex: $(5, -9)$.

Problem 21: Thrown ball

- $h(0) = 6$. **6 feet.**
- $t = -\frac{32}{2(-16)} = 1$, and $h(1) = -16 + 32 + 6 = 22$. **At 1 second, at 22 feet.**
- The graph is symmetric about $t = 1$, and 0 is one second before it, so the other time is $t = 2$. Check: $h(2) = -64 + 64 + 6 = 6$. **At 2 seconds.**



Height of a ball (feet) against time (seconds).

Problem 22: Garden fence

- The third side is $40 - 2x$, so $A(x) = x(40 - 2x) = -2x^2 + 40x$.

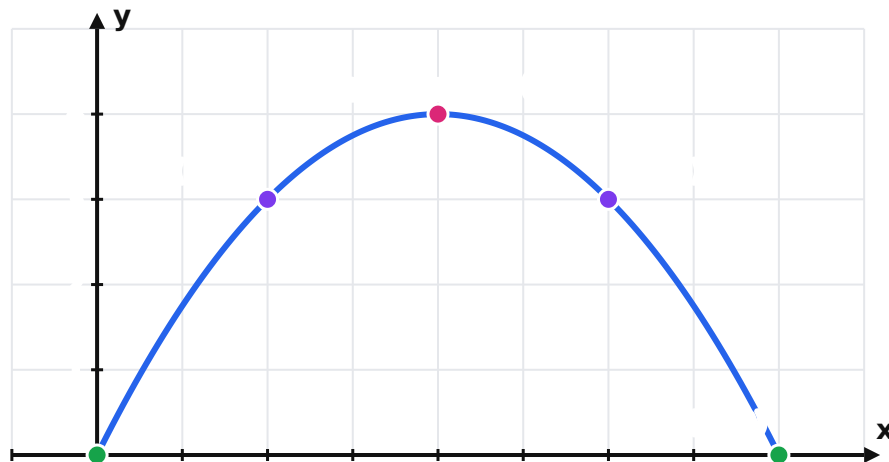
- b. The axis is $x = -\frac{40}{2(-2)} = 10$. $x = 10$ **feet**.
 c. $A(10) = 10(40 - 20) = 200$. The garden is **10 ft by 20 ft**, with area **200 square feet**.

Problem 23: T-shirt sale

- a. $1200 - 60x + 100x - 5x^2 = -5x^2 + 40x + 1200$.
 b. $x = -\frac{40}{2(-5)} = 4$, so the price is $12 + 4 = \$16$.
 c. Shirts sold: $100 - 5(4) = 80$. Revenue: $16 \times 80 = 1280$. **80 shirts, \$1,280**. Check with the formula: $-80 + 160 + 1200 = 1280$.

Problem 24: Fountain arc

- a. $x = -\frac{2}{2(-1/4)} = 4$ and $y = -4 + 8 = 4$. **4 feet**, at $x = 4$.
 b. Solve $y = 0$: $-\frac{1}{4}x(x - 8) = 0$, so $x = 0$ or $x = 8$. The water lands **8 feet** from the nozzle.
 c. $y(2) = -1 + 4 = 3$, so **3 feet**. By symmetry about $x = 4$, the mirror point is $x = 6$; check $y(6) = -9 + 12 = 3$. $x = 6$.



Path of a water stream: x is horizontal distance and y is height, both in feet.