

Algebra 1 · Adding, Subtracting and Multiplying Polynomials · Lesson

Common Core A-APR.A.1, A-SSE.A.2 · mathovore.fr

A polynomial is a sum of terms, each a number times a variable with a whole-number exponent, like $3x^2 - 5x + 7$. You add or subtract polynomials by combining like terms and multiply them by distributing every term, so the result is always another polynomial.

Algebra 1 Common Core: A-APR.A.1 · A-SSE.A.2

Key facts:

- The degree of a polynomial is its largest exponent; standard form lists terms from highest to lowest power.
- To subtract polynomials, change the sign of every term in the second one, then combine like terms.
- FOIL and the box method are two ways to record the same four products of two binomials.
- Special products: $(a+b)^2 = a^2 + 2ab + b^2$, $(a-b)^2 = a^2 - 2ab + b^2$, $(a+b)(a-b) = a^2 - b^2$.
- Polynomials are closed under addition, subtraction and multiplication.

What is a polynomial?

A **polynomial** is a sum of terms. Each term is a number times a variable raised to a whole-number exponent, such as $4x^3$ or -9 . Variables never appear in a denominator or under a radical.

Vocabulary

Coefficient: the number multiplying the variable part of a term.

Degree: the largest exponent in a polynomial. A degree-2 polynomial is a quadratic.

Standard form: terms written from the highest power to the lowest.

Monomial, binomial, trinomial: polynomials with one, two or three terms.

$$3x^2$$

degree 2
coefficient 3

$$-5x$$

degree 1
coefficient -5

$$7$$

degree 0
constant term

The polynomial $3x^2 - 5x + 7$ has three terms. Its degree is 2, the largest exponent.

Polynomials are **closed** under addition, subtraction and multiplication: the result is always another polynomial, just as adding two integers always gives an integer.

How do you add and subtract polynomials?

1. Write each polynomial in standard form.
2. When subtracting, distribute the negative sign to every term of the second polynomial.
3. Combine like terms, which are terms with the same variable and exponent.

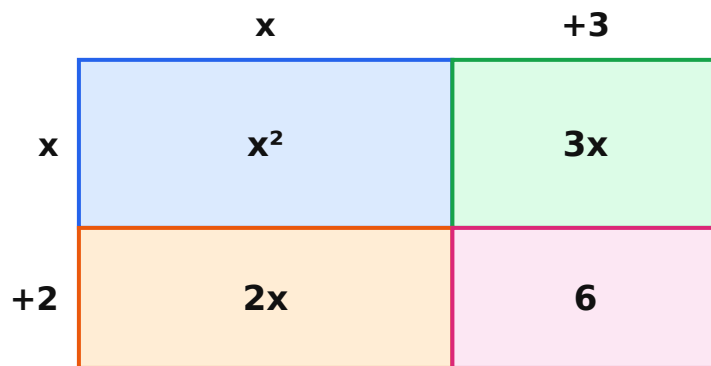
Worked example. $(5x^2 + 2x - 7) - (2x^2 - 3x + 4)$ becomes $5x^2 + 2x - 7 - 2x^2 + 3x - 4$, which simplifies to $3x^2 + 5x - 11$.

	x^2 terms	x terms	constants
first	5	2	-7
negated	-2	+3	-4
sum	3	5	-11

Subtracting polynomials: change every sign in the second polynomial, then add each column of like terms.

How do you multiply polynomials?

Multiply every term of the first polynomial by every term of the second, then combine like terms. For two binomials, **FOIL** lists the First, Outer, Inner and Last products. The **box method** draws the same four products as an area model.



The box method for $(x+3)(x+2)$: add the four cells to get x^2+5x+6 .

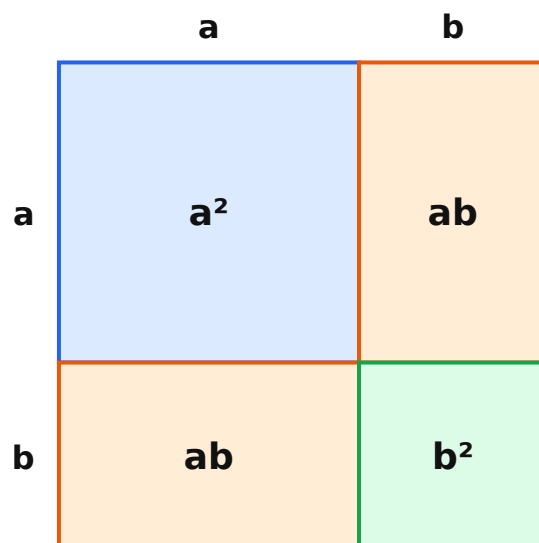
Worked example. To multiply $(2x-5)(x+6)$ with FOIL, find First $2x^2$, Outer $12x$, Inner $-5x$ and Last -30 . Combine the Outer and Inner terms: $12x-5x=7x$. The product is $2x^2+7x-30$. Always check one value: at $x=1$ both forms equal -21 .

Common mistakes. Students often forget to distribute a negative sign when subtracting, add exponents when they should add coefficients, or write $(x+4)^2 = x^2+16$ and lose the middle term.

What are the special products?

Three patterns save time and appear again when factoring.

Pattern	Result
$(a+b)^2$	$a^2+2ab+b^2$
$(a-b)^2$	$a^2-2ab+b^2$
$(a+b)(a-b)$	a^2-b^2



A square with side $a+b$ has area $(a+b)^2 = a^2 + 2ab + b^2$.

Examples. $(2x+5)^2 = 4x^2 + 20x + 25$. Mental math also works:
 $51 \times 49 = (50+1)(50-1) = 2500 - 1 = 2499$.

Adding, Subtracting and Multiplying Polynomials: frequently asked questions

How do you find the degree of a polynomial?

Look at each term's exponent and pick the largest one. In $4x^5 - x^2 + 9$ the degree is 5. For a term with two variables, such as $3x^2y$, add the exponents to get degree 3.

What are like terms?

Like terms have exactly the same variable part, including the same exponents. $6x^2$ and $-2x^2$ are like terms, but $6x^2$ and $6x$ are not.

Why is $(x + 4)^2$ not equal to $x^2 + 16$?

Squaring means multiplying the binomial by itself: $(x+4)(x+4) = x^2 + 8x + 16$. The middle term $8x$ comes from the two Outer and Inner products. Testing $x=1$ gives 25 on the left but only 17 on the right.

What is the difference between FOIL and the box method?

Both find the same four products. FOIL lists them in a line in the order First, Outer, Inner, Last. The box method places them in a grid, which also works for polynomials with more than two terms.

What does it mean that polynomials are closed?

Adding, subtracting or multiplying two polynomials always produces another polynomial. Division is different: $x \div (x+1)$ is not a polynomial, so polynomials are not closed under division.