

Algebra 1 · Linear and Compound Inequalities · Lesson

Common Core A-REI.B.3, A-CED.A.1, A-CED.A.3 · mathovore.fr

A linear inequality is solved like an equation, except that multiplying or dividing both sides by a negative number reverses the inequality sign. The solution is a set of numbers shown on a number line with open or closed circles. Compound inequalities join two conditions with and, giving an overlap, or or, giving a union.

Algebra 1 Common Core: A-REI.B.3 · A-CED.A.1 · A-CED.A.3

Key facts:

- Standards A-REI.B.3, A-CED.A.1 and A-CED.A.3 cover solving, writing and modeling with linear inequalities.
- Reverse the inequality sign when multiplying or dividing both sides by a negative number.
- Open circle or parenthesis excludes an endpoint; closed circle or bracket includes it.
- And means the overlap of the two solution sets; or means their union.
- In context, check whether the solution must be a whole number.

What is a linear inequality?

A **linear inequality** compares two expressions with $<$, $>$, $\leq$ or $\geq$. Its solution is usually a whole set of numbers, not one number. The graph on a number line shows the set: an **open circle** means the endpoint is not included, a **closed circle** means it is.

Three ways to write the same solution set

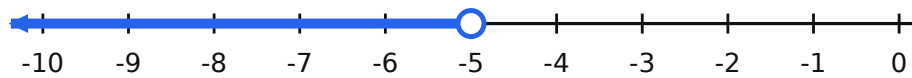
Inequality	Interval notation	Graph
$x < -5$	$(-\infty, -5)$	open circle, ray left
$x \geq -4$	$[-4, \infty)$	closed circle, ray right
$-2 < x \leq 3$	$(-2, 3]$	open and closed circles joined

How do you solve a multi-step inequality?

1. Distribute and combine like terms on each side.

2. Move variable terms to one side and constants to the other with addition or subtraction.
3. Divide or multiply by the coefficient. **If it is negative, reverse the inequality sign.**
4. Graph the solution and test one point from the shaded set.

Worked example. Solve $-3x+5>20$. Subtract 5: $-3x>15$. Divide by -3 and flip the sign: $x<-5$. Test $x=-6$: $-3(-6)+5=23$, and $23>20$ is true.



The solution $x<-5$ has an open circle at -5 .

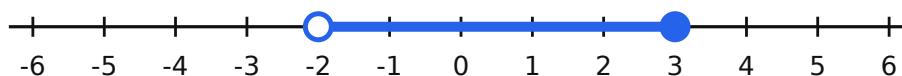
Why flip? Multiplying by a negative reverses order: $2<5$ but $-2>-5$.

Second example. Solve $2(x-3)\leq 4x+2$. Distribute: $2x-6\leq 4x+2$. Subtract $4x$: $-2x-6\leq 2$. Add 6: $-2x\leq 8$. Divide by -2 and reverse the sign: $x\geq -4$. The graph has a closed circle at -4 and a ray to the right.

Check your work. Pick one number inside the shaded region and one outside, then substitute both into the original inequality. The inside number must make it true and the outside number must make it false. This quick test catches a forgotten sign reversal, the most common mistake on this topic.

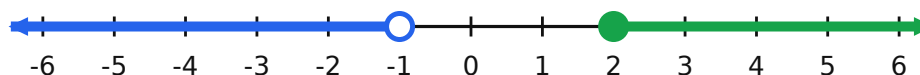
What are compound inequalities?

An **and** inequality needs both parts true, so its graph is the overlap, often a segment. Solve $-1<2x+3\leq 9$ by working on all three parts: subtract 3 to get $-4<2x\leq 6$, divide by 2 to get $-2<x\leq 3$.



Graph of $-2<x\leq 3$.

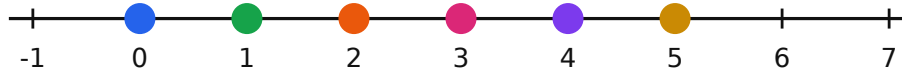
An **or** inequality is true when at least one part is true, so its graph is the union, often two rays: $x<-1$ or $x\geq 2$.



Graph of $x < -1$ or $x \geq 2$.

How do you model a constraint with an inequality?

Phrases translate directly: *at most* means $\leq$, *at least* means $\geq$, *fewer than* means $<$. Maya has \$40 for T-shirts at \$6 each plus a \$7 bag. Then $6t + 7 \leq 40$, so $t \leq 5.5$. Because shirts come in whole numbers, she can buy at most 5.



Only whole numbers 0 to 5 make sense for the number of shirts.

Linear and Compound Inequalities: frequently asked questions

Why do you flip the inequality sign when dividing by a negative number?

Multiplying by a negative number reverses the order of numbers. Since $2 < 5$ but $-2 > -5$, the sign must flip to keep the statement true. Test a point to confirm.

When do I use an open circle and when a closed circle?

Use an open circle for $<$ or $>$ because the endpoint is not a solution. Use a closed circle for $\leq$ or $\geq$ because the endpoint is included.

What is interval notation?

It lists the endpoints of the solution set. Parentheses exclude an endpoint and brackets include it, so $-2 < x \leq 3$ is written $(-2, 3]$. Infinity always gets a parenthesis.

What is the difference between and and or in a compound inequality?

With and, a number must satisfy both inequalities, so the graph is the overlap. With or, a number needs to satisfy at least one, so the graph is both pieces together.

How do I solve an inequality with no variables left?

If the variable cancels and the statement is true, such as $6 > 1$, every real number is a solution. If the statement is false, such as $5 < -1$, there is no solution.