

Algebra 1 · Linear Functions: Slope-Intercept, Point-Slope and Standard Form · Lesson

Common Core F-IF.C.7a, F-LE.A.1a, F-LE.A.2, A-CED.A.2 · mathovore.fr

A linear function has a constant rate of change called its slope. It can be written as $y = mx + b$ (slope-intercept), $y - y_1 = m(x - x_1)$ (point-slope) or $Ax + By = C$ (standard). Find the slope from two points, graph the line from any form, and write models from tables, graphs or words.

Algebra 1 Common Core: F-IF.C.7a · F-LE.A.1a · F-LE.A.2 · A-CED.A.2

Key facts:

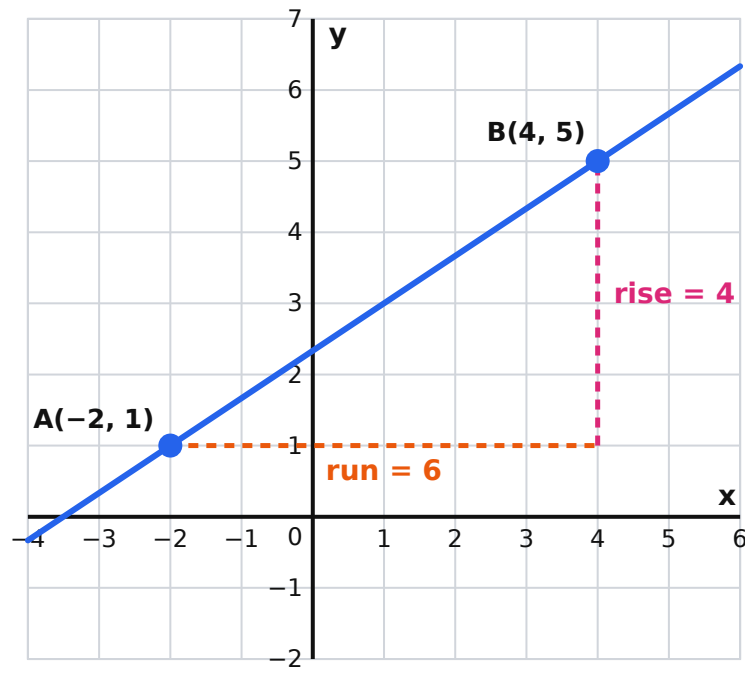
- Standards F-IF.C.7a, F-LE.A.1a, F-LE.A.2 and A-CED.A.2: graph linear functions, recognize constant rates of change, and write linear models.
- Slope from two points: $m = \frac{y_2 - y_1}{x_2 - x_1}$.
- Slope-intercept $y = mx + b$, point-slope $y - y_1 = m(x - x_1)$, standard $Ax + By = C$.
- Parallel lines have equal slopes; perpendicular lines have slopes whose product is -1 .
- In a model, the slope is the rate of change and the y-intercept is the starting value.

What is slope and how do you find it?

A linear function changes by the same amount every time x goes up by 1. That constant rate of change is the **slope**. Given two points (x_1, y_1) and (x_2, y_2) , subtract the coordinates in the same order:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For $(-2, 1)$ and $(4, 5)$, $m = \frac{5 - 1}{4 - (-2)} = \frac{4}{6} = \frac{2}{3}$. A positive slope rises from left to right, a negative slope falls, a horizontal line has slope 0, and a vertical line has undefined slope.



Slope is rise over run: from $A(-2,1)$ to $B(4,5)$ the line rises 4 while it runs 6, so $m = \frac{4}{6} = \frac{2}{3}$.

What are the three forms of a linear equation?

Forms of a line

Slope-intercept form: $y = mx + b$, where m is the slope and b is the y-intercept.

Point-slope form: $y - y_1 = m(x - x_1)$, for a line with slope m through (x_1, y_1) .

Standard form: $Ax + By = C$, usually with integer coefficients and $A \geq 0$.

Slope-intercept

$$y = mx + b$$

Point-slope

$$y - y_1 = m(x - x_1)$$

Standard

$$Ax + By = C$$

The three forms describe the same line; each one makes a different feature easy to see.

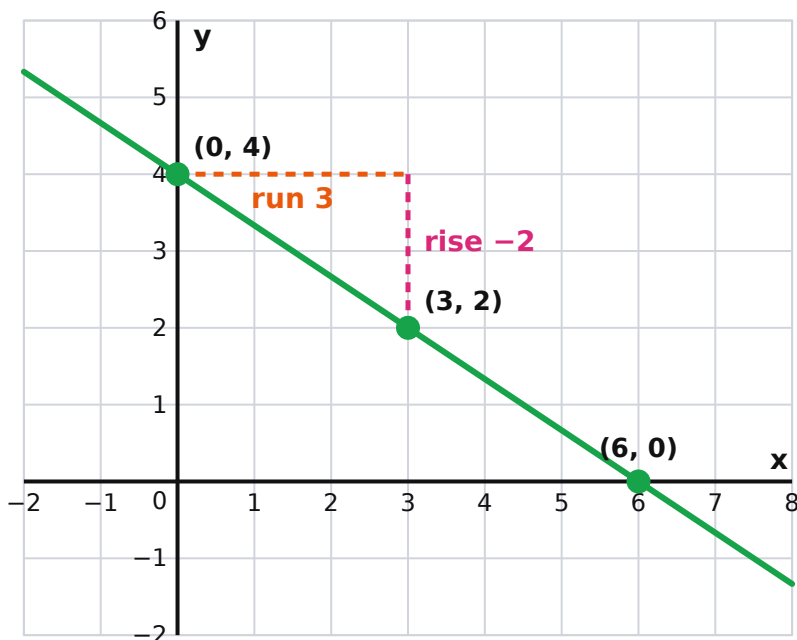
To convert, solve for y or clear fractions. For example, $2x + 3y = 12$ becomes $3y = -2x + 12$, so $y = -\frac{2}{3}x + 4$.

How do you graph a line from its equation?

1. Write the equation in a form that shows a point and the slope.
2. Plot that point, such as the y-intercept $(0, b)$.

3. Use rise over run to plot at least one more point, then draw the line through them.
4. Check a third point in the equation.

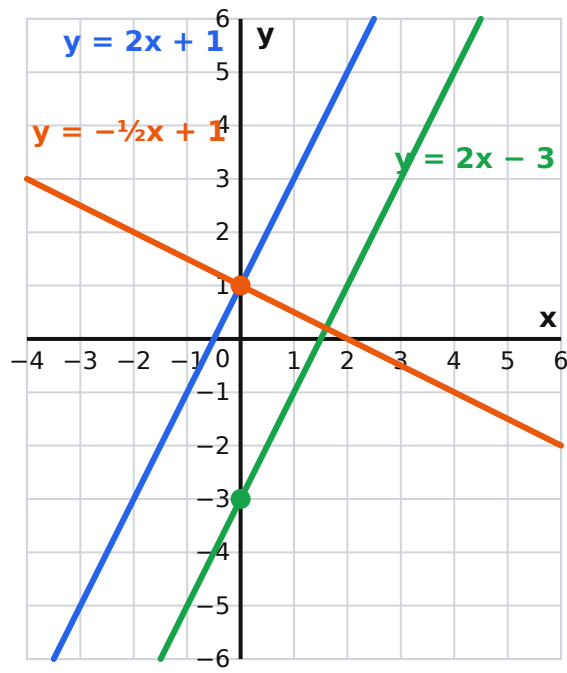
Worked example. Graph $y = -\frac{2}{3}x + 4$. Plot $(0, 4)$. The slope $-\frac{2}{3}$ means down 2 and right 3, which gives $(3, 2)$ and $(6, 0)$. Check: $-\frac{2}{3}(6) + 4 = 0$.



Graphing $y = -\frac{2}{3}x + 4$: plot the y-intercept $(0, 4)$, then go right 3 and down 2 to reach $(3, 2)$ and again to reach $(6, 0)$.

How are parallel and perpendicular lines related?

Parallel lines have equal slopes and different y-intercepts. Perpendicular lines have slopes that are negative reciprocals, so their product is -1 . If $m = 2$, a perpendicular line has slope $-\frac{1}{2}$.



The two blue and green lines share slope 2, so they are parallel. The orange line has slope $-\frac{1}{2}$, the negative reciprocal of 2, so it is perpendicular to both.

How do you write a linear model from a situation?

Find the constant rate of change (the slope) and the starting value (the y-intercept). A gym that charges a \$25 sign-up fee plus \$18 per month has cost $C = 18m + 25$. Two points from a table also work: find m first, then use point-slope form.

Linear Functions: Slope-Intercept, Point-Slope and Standard Form: frequently asked questions

How do you find the slope of a line from two points?

Divide the change in y by the change in x, keeping the same point first in both differences. For (1,3) and (5,11), $m = \frac{11-3}{5-1} = 2$.

Which form of a linear equation should I use?

Use slope-intercept form to graph quickly from the y-intercept and slope. Use point-slope form when you know a point and the slope. Use standard form to find intercepts or to write equations with integer coefficients.

How do you convert standard form to slope-intercept form?

Solve the equation for y. For $3x + 4y = 12$, subtract $3x$ to get $4y = -3x + 12$, then divide by 4 to

get $y = -\frac{3}{4}x + 3$.

How do you write a line parallel or perpendicular to a given line?

A parallel line keeps the same slope. A perpendicular line uses the negative reciprocal of the slope. Then plug the given point into point-slope form.

What do slope and y-intercept mean in a word problem?

The slope is the amount the output changes per one unit of input, such as dollars per month. The y-intercept is the starting amount when the input is 0, such as a sign-up fee.