

Algebra 1 · Linear Equations and Literal Equations · Lesson

Common Core A-REI.A.1, A-REI.B.3, A-CED.A.1, A-CED.A.4 · mathovore.fr

A linear equation in one variable can be solved by a chain of justified steps: distribute, combine like terms, clear fractions, move variable terms to one side, and divide by the coefficient. Each step uses a property of equality, and the answer is checked in the original equation. Literal equations use the same steps for formulas.

Algebra 1 Common Core: A-REI.A.1 · A-REI.B.3 · A-CED.A.1 · A-CED.A.4

Key facts:

- Standards A-REI.A.1, A-REI.B.3, A-CED.A.1 and A-CED.A.4 cover justified solving, multi-step equations, writing equations and rearranging formulas.
- Every step must be backed by a property: equality properties, the distributive property, or combining like terms.
- Multiply by the least common denominator to clear fractions before solving.
- If the variable cancels, a false statement means no solution and a true statement means infinitely many solutions.
- To solve a formula for one letter, treat the other letters as numbers: for $A = (1/2)bh$, $h = 2A/b$.

What does it mean to justify a step in solving an equation?

To **solve** a linear equation, you rewrite it as a chain of simpler equations that all have the same solution. Each move must be backed by a reason, such as a property of equality or the distributive property. If every step has a reason, the final answer is guaranteed to satisfy the original equation.

Properties that justify steps

Addition and subtraction properties of equality: adding or subtracting the same number on both sides keeps the equation true.

Multiplication and division properties of equality: multiplying or dividing both sides by the same nonzero number keeps it true.

Distributive property: $a(b+c) = ab+ac$. **Combining like terms** adds the coefficients of matching variable terms.

How do you solve a multi-step linear equation?

1. Clear parentheses with the distributive property and combine like terms on each side.
2. Clear fractions or decimals by multiplying every term by the least common denominator or a power of 10.
3. Collect the variable terms on one side and the constants on the other, using inverse operations.
4. Divide by the coefficient, then check the answer in the original equation.

Worked example. Solve $3(x+2)=2x+11$. Distribute: $3x+6=2x+11$. Subtract $2x$: $x+6=11$. Subtract 6: $x=5$. Check: $3(7)=21$ and $2(5)+11=21$.

$3(x + 2) = 2x + 11$	→ Given
$3x + 6 = 2x + 11$	→ Distributive property
$x + 6 = 11$	→ Subtract $2x$ from both sides
$x = 5$	→ Subtract 6 from both sides

Each line is the previous line with one move, and each move has a reason.

With fractions. To solve $\frac{x}{2} + \frac{x}{3} = 10$, multiply every term by 6 to get $3x+2x=60$, so $x=12$.

Can a linear equation have no solution or infinitely many?

Yes. Simplify until the variable cancels. A false statement such as $2=5$ means **no solution**. A true statement such as $2=2$ means **infinitely many solutions**: every real number works.

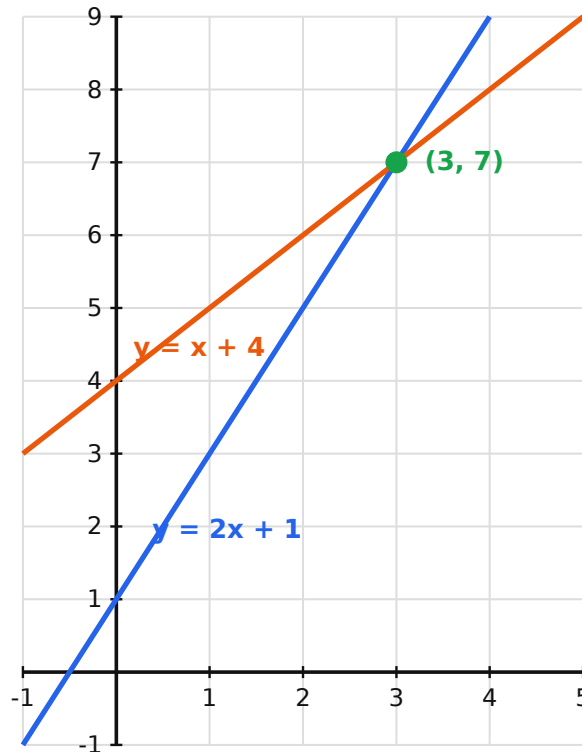
One solution $3x + 1 = 10$ $x = 3$	No solution $x + 2 = x + 5$ $2 = 5$ false	Infinitely many $2(x + 1) = 2x + 2$ $2 = 2$ true
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Simplifying until the variable disappears shows whether there is no solution or infinitely many.

How does a graph show the solution of an equation?

Graph the left side and the right side as two lines. The x -coordinate where they cross is the

solution. For $2x+1=x+4$ the lines meet at $(3,7)$, so $x=3$.



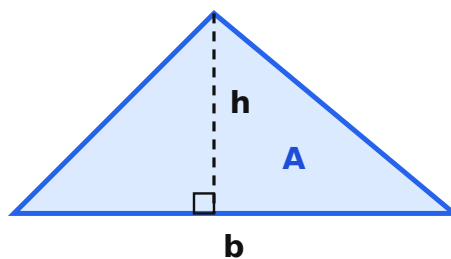
Graphing each side as a line: the lines cross where $2x+1=x+4$, at $x=3$.

How do you write an equation from a word problem?

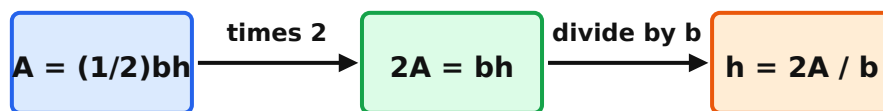
Name the unknown, then translate each phrase. A taxi charges a \$3.50 fee plus \$2.25 per mile. If the ride costs \$17.00, then $3.50+2.25m=17$, so $m=6$ miles.

What is a literal equation?

A **literal equation** is a formula with several letters. To solve for one letter, treat the others as numbers and use inverse operations. For $A = \frac{1}{2}bh$, multiply both sides by 2 to get $2A = bh$, then divide by b : $h = \frac{2A}{b}$.



A triangle with base b , height h and area A .



Solving $A = \frac{1}{2}bh$ for h takes two inverse operations.

Linear Equations and Literal Equations: frequently asked questions

How do I solve an equation with variables on both sides?

Move the variable terms to one side by adding or subtracting the same term on both sides. For $7x - 3 = 5x + 11$, subtract $5x$ to get $2x - 3 = 11$, add 3, then divide by 2 to find $x = 7$.

How do I get rid of fractions in an equation?

Multiply every term on both sides by the least common denominator. In $\frac{x}{2} + \frac{x}{3} = 10$ the denominator is 6, which gives $3x + 2x = 60$ and $x = 12$.

What does it mean when an equation has no solution?

It means no number makes both sides equal. After simplifying, you get a false statement such as $6 = 5$. The two sides, graphed as lines, are parallel and never meet.

What is the difference between an identity and an equation with one solution?

An identity such as $3(x+2) = 3x+6$ is true for every value of x , so it has infinitely many solutions. An equation with one solution is true for exactly one value.

How do I solve a formula for a different variable?

Use the same inverse operations as for numbers, in reverse order of operations. To solve $P = 2l + 2w$ for w , subtract $2l$ and divide by 2, giving $w = \frac{P-2l}{2}$.