

Algebra 1 · Key Features of Graphs and Rate of Change · Worksheet Answer Key

Common Core F-IF.B.4, F-IF.B.6, F-IF.C.9 · mathovore.fr

Each solution shows the work, with the final result in bold. When your interval or point differs, check the order of subtraction in rate-of-change problems, and check that you used strict inequalities for open intervals.

Algebra 1 Common Core: F-IF.B.4 · F-IF.B.6 · F-IF.C.9

Problem 1: Intercepts of a line

For the y -intercept, $f(0) = 2(0) - 6 = -6$.

For the x -intercept, $2x - 6 = 0$, so $2x = 6$ and $x = 3$.

y -intercept $(0, -6)$; **x -intercept** $(3, 0)$

Problem 2: Intercepts of a parabola

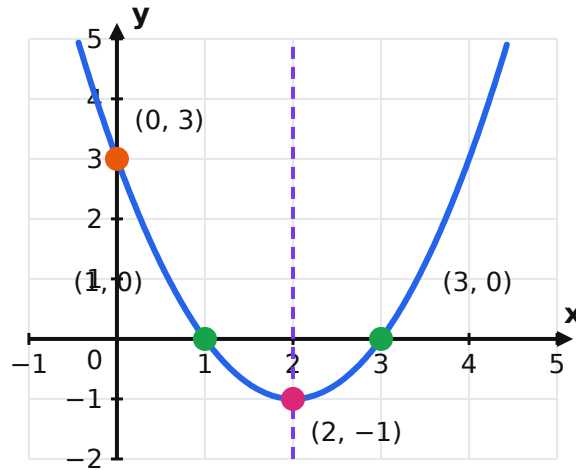
$g(0) = -9$, so the y -intercept is $(0, -9)$.

Solve $x^2 - 9 = 0$: $x^2 = 9$, so $x = 3$ or $x = -3$.

y -intercept $(0, -9)$; **x -intercepts** $(-3, 0)$ and $(3, 0)$

Problem 3: Features from a graph

The graph meets the x -axis at 1 and 3 and the y -axis at 3. The lowest point is $(2, -1)$. The axis of symmetry is the vertical line through the vertex; it is halfway between the intercepts 1 and 3.



Intercepts $(1,0)$, $(3,0)$, $(0,3)$; vertex $(2,-1)$; axis of symmetry $x=2$.

x-intercepts $(1,0)$, $(3,0)$; **y-intercept** $(0,3)$; **vertex** $(2,-1)$, **a minimum**; **axis** $x=2$

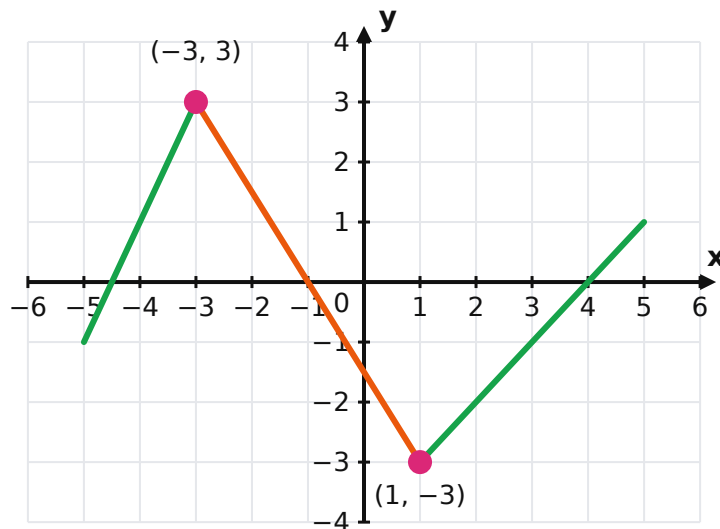
Problem 4: Intercepts from a table

The column $x=0$ gives $f(0)=12$. The column with output 0 is $x=3$. The outputs drop by 4 each step.

y-intercept $(0,12)$; **x-intercept** $(3,0)$; **decreasing**

Problem 5: Increasing and decreasing

Reading left to right, the graph rises to $(-3,3)$, falls to $(1,-3)$, then rises again.



Green: increasing. Orange: decreasing. Pink points: relative maximum $(-3,3)$ and relative minimum $(1,-3)$.

Increasing: $-5 < x < -3$ and $1 < x < 5$. **Decreasing:** $-3 < x < 1$. **Relative maximum 3 at** $x = -3$

; **relative minimum** -3 at $x=1$.

Problem 6: Positive and negative intervals

The graph meets the x -axis at -4 , 0 and 4 . Between -4 and 0 it is above the axis; between 0 and 4 it is below the axis.

x -intercepts -4 , 0 , 4 . **Positive:** $-4 < x < 0$. **Negative:** $0 < x < 4$.

Problem 7: Axis of symmetry and vertex

The axis is $x = -\frac{b}{2a} = -\frac{-6}{2(1)} = 3$. Then $f(3) = 9 - 18 + 5 = -4$, so the vertex is $(3, -4)$.

The y -intercept is $(0, 5)$. It is 3 units left of the axis, so the mirror point is 3 units right: $(6, 5)$.
Check: $f(6) = 36 - 36 + 5 = 5$.

Axis $x=3$; **vertex** $(3, -4)$; **symmetric point** $(6, 5)$

Problem 8: Features of an absolute value function

The vertex is where $x-2=0$: $(2, -3)$, a minimum. The y -intercept is $f(0) = 2 - 3 = -1$. For x -intercepts, $|x-2|=3$, so $x-2=3$ or $x-2=-3$, giving $x=5$ or $x=-1$.

Vertex $(2, -3)$; **y -intercept** $(0, -1)$; **x -intercepts** $(-1, 0)$, $(5, 0)$; **decreasing for** $x < 2$; **increasing for** $x > 2$; **symmetric about** $x=2$

Problem 9: Maximum of a quadratic

The axis is $x = -\frac{4}{2(-1)} = 2$ and $f(2) = -4 + 8 + 1 = 5$. The parabola opens down, so the vertex $(2, 5)$ is a maximum. The y -intercept is $f(0) = 1$.

Vertex $(2, 5)$; **maximum value** 5 ; **y -intercept** $(0, 1)$; **increasing for** $x < 2$; **decreasing for** $x > 2$

Problem 10: End behavior

- Large x : the outputs fall without bound. Very negative x : the outputs rise without bound.
Falls on the right, rises on the left.
- Large or very negative x : x^2 is large, so $g(x)$ rises without bound at both ends. **Rises on both ends.**
- Large x : 3^x grows without bound. Very negative x : 3^x is a tiny positive number, like $\frac{1}{27}$ for $x = -3$. **Grows on the right, approaches 0 on the left.**

Problem 11: Average rate of change: squares

$f(1)=1$ and $f(4)=16$. Then $\frac{16-1}{4-1}=\frac{15}{3}=5$.

5

Problem 12: A rate of change of zero

$f(-1)=1+2=3$ and $f(3)=9-6=3$. So $\frac{3-3}{3-(-1)}=\frac{0}{4}=0$.

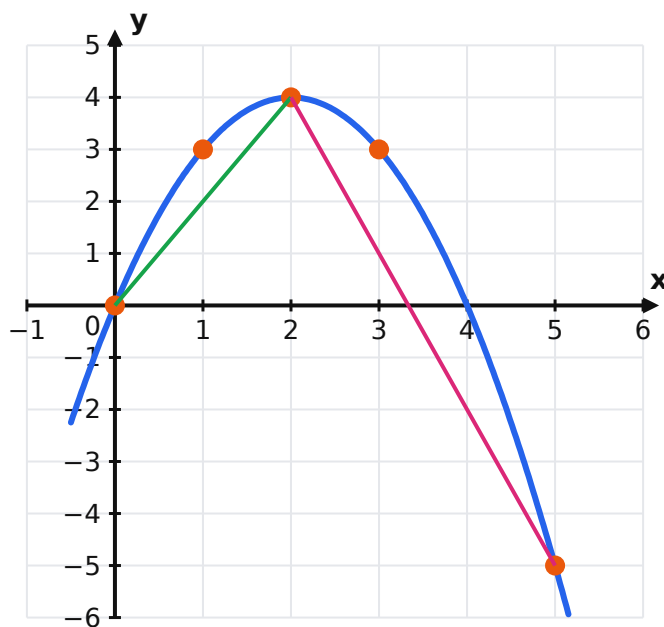
The function goes down and then up, so the changes cancel; the starting and ending outputs are equal.

0

Problem 13: Average rate of change from a graph

Read the points: $f(0)=0$, $f(1)=3$, $f(2)=4$, $f(3)=3$, $f(5)=-5$.

$\frac{4-0}{2-0}=2$; $\frac{-5-4}{5-2}=\frac{-9}{3}=-3$; $\frac{3-3}{3-1}=0$.



Green secant: $x=0$ to $x=2$, slope 2. Pink secant: $x=2$ to $x=5$, slope -3 .

2, -3 and 0

Problem 14: Rate of change of an exponential

$f(1)=2$, $f(3)=8$, $f(5)=32$.

$$\frac{8-2}{3-1} = \frac{6}{2} = 3 \text{ and } \frac{32-8}{5-3} = \frac{24}{2} = 12.$$

The rate of change is four times as large on the later interval, so the function grows faster and faster.

3 and 12

Problem 15: Equation versus table

For f : rate 3, y -intercept -2 . For g : the outputs rise 4 per step, so the rate is 4, and $g(0) = 1$.

At $x = 10$: $f(10) = 28$, and $g(10) = 1 + 4(10) = 41$.

g has the greater rate (4 vs 3), the greater y -intercept (1 vs -2), and the greater value at 10 (41 vs 28).

Problem 16: Equation versus graph

For f : the vertex is $(0, 9)$, so the maximum is 9. $f(0) = 9$. Solve $-x^2 + 9 = 0$: $x = \pm 3$.

For g : the maximum is the vertex height, 4. The y -intercept is -5 . The x -intercepts are 1 and 5.

f has the greater maximum (9 vs 4) and the greater y -intercept (9 vs -5). f : $x = -3, 3$; g : $x = 1, 5$.

Problem 17: Ball height features

$h(0) = 5$: the ball starts 5 feet above the ground.

The axis is $t = -\frac{64}{2(-16)} = 2$, and $h(2) = -64 + 128 + 5 = 69$.

$\frac{h(2) - h(0)}{2 - 0} = \frac{69 - 5}{2} = 32$ feet per second.

Starts at 5 ft; highest at 2 s, 69 ft; average rate 32 ft/s

Problem 18: Draining a tank

$V(0) = 120$: the tank starts with 120 gallons. Solve $120 - 8t = 0$: $t = 15$, so the tank is empty after 15 minutes.

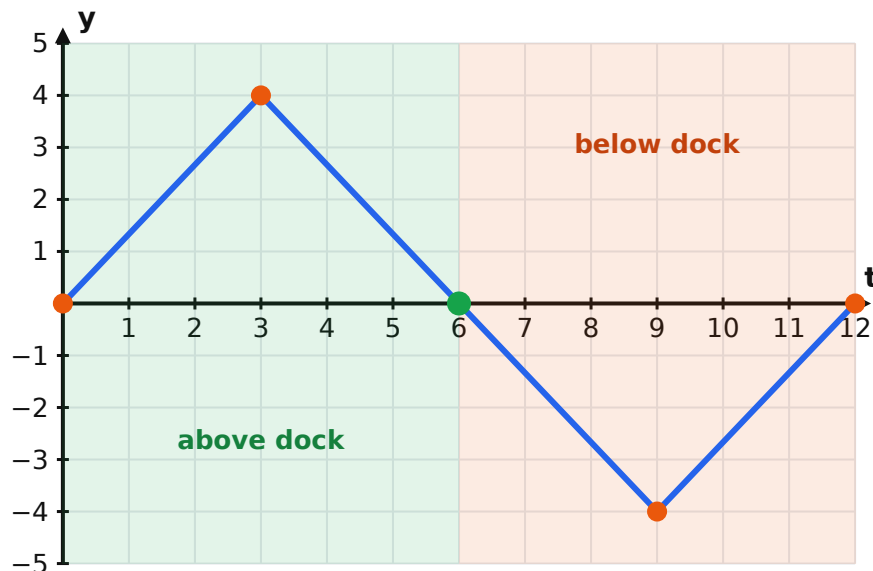
The rate of change is -8 : the tank loses 8 gallons every minute.

V -intercept 120 gal; t -intercept 15 min; rate -8 gal/min; domain $0 \leq t \leq 15$

Problem 19: Tide chart

The graph is above the axis from $t = 0$ to $t = 6$. It rises from 0 to 3 hours and from 9 to 12 hours. The peak is $(3, 4)$ and the lowest point is $(9, -4)$.

$$\frac{-4 - 4}{9 - 3} = \frac{-8}{6} = -\frac{4}{3} \text{ feet per hour.}$$



The level is above the dock for $0 < t < 6$ (green) and below it for $6 < t < 12$ (orange).

Above dock: $0 < t < 6$. **Increasing:** $0 < t < 3$ and $9 < t < 12$. **Maximum 4 ft at 3 h; minimum -4 ft at 9 h. Average rate $-\frac{4}{3}$ ft/h.**

Problem 20: Road trip speeds

$$\frac{230-0}{4-0} = 57.5 \text{ mph. } \frac{180-62}{3-1} = \frac{118}{2} = 59 \text{ mph. } \frac{230-118}{4-2} = \frac{112}{2} = 56 \text{ mph.}$$

57.5 mph, 59 mph and 56 mph; the lowest speed was from $t = 2$ to $t = 4$.

Problem 21: Gym memberships

Gym A has rate \$20 per month (plus a \$50 fee). Gym B rises \$45 per month, so $C_B(m) = 45m$.

After 1 month: $A = 70$, $B = 45$. After 2 months: $A = 90$, $B = 90$. After 6 months: $A = 20(6) + 50 = 170$, $B = 45(6) = 270$.

B has the greater rate (45 vs 20). Gym A is cheaper after more than 2 months, the same at 2 months, and costlier for 1 month.

Problem 22: Exponential versus linear growth

From 0 to 4: $F: \frac{16-1}{4} = 3.75$; $G: \frac{40-0}{4} = 10$.

From 0 to 6: $F: \frac{64-1}{6} = 10.5$; $G: \frac{60-0}{6} = 10$.

On 0 to 4, G is greater (10 vs 3.75). On 0 to 6, F is greater (10.5 vs 10): the exponential overtakes the line.