

Algebra 1 · Key Features of Graphs and Rate of Change · Lesson

Common Core F-IF.B.4, F-IF.B.6, F-IF.C.9 · mathovore.fr

Key features of a graph include its intercepts, where it increases or decreases, relative maximum and minimum points, end behavior and symmetry. The average rate of change from a to b is $(f(b) - f(a)) / (b - a)$, the slope of the secant line.

Algebra 1 Common Core: F-IF.B.4 · F-IF.B.6 · F-IF.C.9

Key facts:

- The x -intercepts are the inputs where $f(x) = 0$; the y -intercept is $f(0)$.
- Increasing means the graph rises from left to right; decreasing means it falls.
- A relative maximum or minimum is a turning point, higher or lower than nearby points.
- Average rate of change = $(f(b) - f(a)) / (b - a)$, with units.
- To compare functions, compare the same feature, such as rate or intercept.

What are the key features of a graph?

The **key features** of a function are the facts about its graph that tell its story: where it crosses the axes, where it rises and falls, where it turns, and how it behaves at the far left and far right.

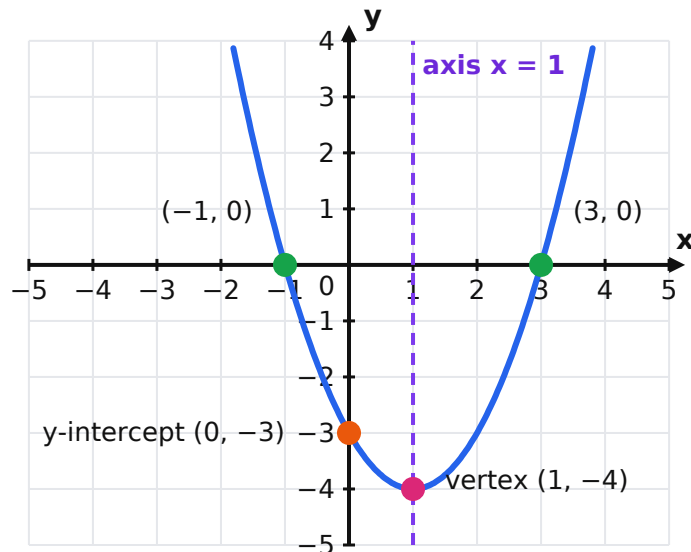
Vocabulary

Intercepts: the x -intercept is where $f(x) = 0$ (the graph meets the x -axis); the y -intercept is the point $(0, f(0))$.

Relative maximum or minimum: a point higher or lower than all the nearby points on the graph.

End behavior: what happens to $f(x)$ as x gets very large or very negative.

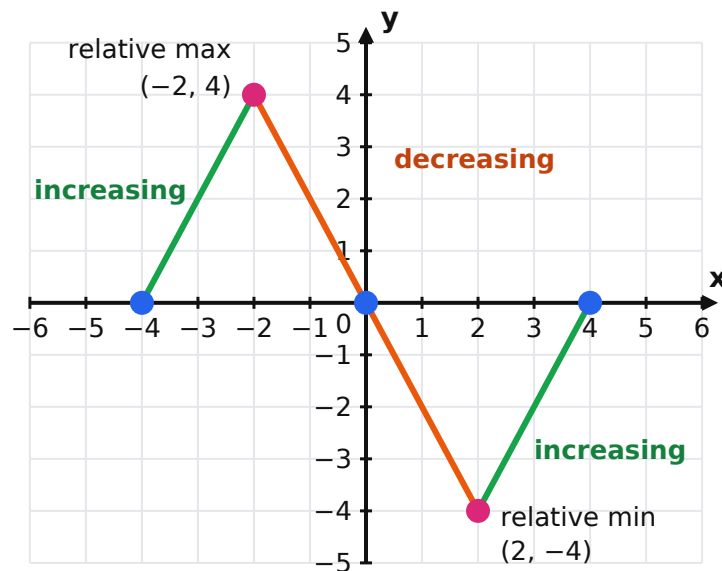
Symmetry: the graph can be folded onto itself along a vertical line, as a parabola can.



Key features of $f(x) = x^2 - 2x - 3$: two x -intercepts, one y -intercept, a vertex, and a vertical axis of symmetry.

How do you read increasing, decreasing, positive and negative?

Read the graph from left to right. A function is **increasing** where the graph rises and **decreasing** where it falls. It is **positive** where the graph is above the x -axis and **negative** where it is below. Describe intervals by their x -values.

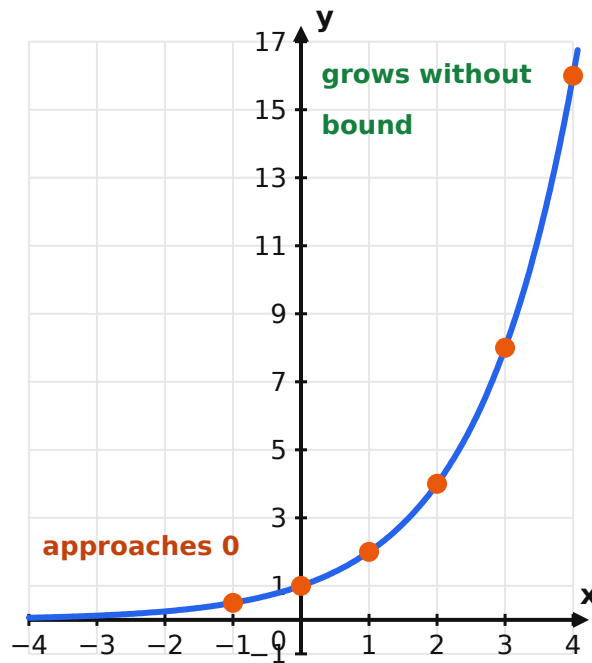


Green parts rise, the orange part falls. The turning points are a relative maximum and a relative minimum.

What are end behavior and symmetry?

For $f(x) = 2^x$, the values grow without bound as x gets large, and they get closer to 0 as x gets

very negative. A parabola opens up or down, so both ends go the same direction, and its axis of symmetry passes through the vertex at $x = -\frac{b}{2a}$.



End behavior of $f(x) = 2^x$: the left end approaches 0, the right end grows without bound.

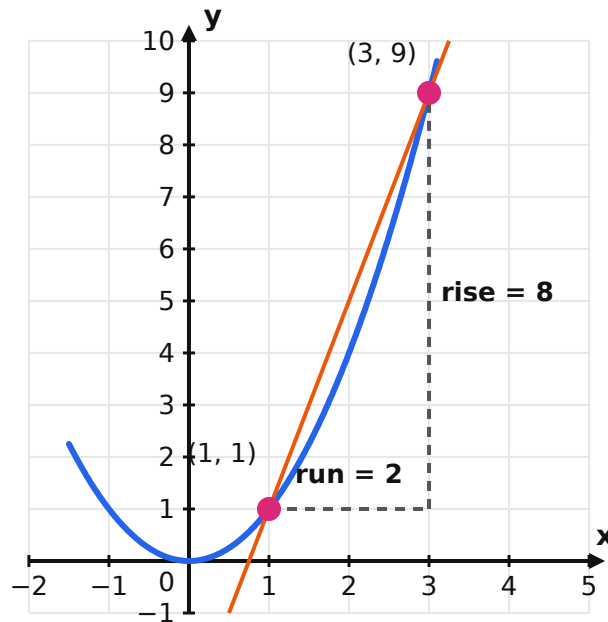
How do you find an average rate of change?

The **average rate of change** of f from $x = a$ to $x = b$ is the change in output divided by the change in input:

$$\frac{f(b) - f(a)}{b - a}$$

1. Find $f(a)$ and $f(b)$.
2. Subtract the outputs, then subtract the inputs, in the same order.
3. Divide, and include units such as miles per hour.

Worked example. For $f(x) = x^2$ from $x = 1$ to $x = 3$: $f(1) = 1$ and $f(3) = 9$, so the rate is $\frac{9-1}{3-1} = \frac{8}{2} = 4$. For a line this number is the slope; for a curve it is the slope of the secant line.



The average rate of change of $f(x) = x^2$ from $x = 1$ to $x = 3$ is the slope of the secant line: $\frac{8}{2} = 4$.

How do you compare two functions?

Compare the same feature for both, such as the rate of change, the y -intercept or the value at a given input. Suppose $f(x) = 3x + 2$ and g is given by a table.

x	0	1	2	3
$g(x)$	5	7	9	11

The table rises 2 per step, so g has rate 2 and y -intercept 5. Function f has rate 3 and y -intercept 2. So f grows faster, but g starts higher.

Key Features of Graphs and Rate of Change: frequently asked questions

What is the difference between an x -intercept and a zero?

They describe the same input. A zero of f is a value of x with $f(x) = 0$, and the x -intercept is the point $(x, 0)$ where the graph crosses the axis.

How do you tell increasing from decreasing?

Read the graph from left to right. If it goes up, the function is increasing on that interval. If it goes down, the function is decreasing there.

What is a relative maximum?

It is a point on the graph that is higher than all the points close to it. It does not have to be the highest point overall. A relative minimum is the matching low point.

Does average rate of change work for curves?

Yes. It gives the slope of the straight line joining two points on the curve. For a line it equals the slope everywhere, but for a curve it depends on the interval you choose.

How can I compare a function given by a table and one given by an equation?

Find the same feature for each, such as the rate of change from two table rows and the slope from the equation. Then compare the numbers, including units.