

Algebra 1 · Factoring Polynomials · Lesson

Common Core A-SSE.A.2, A-SSE.B.3a · mathovore.fr

Factoring rewrites a polynomial as a product of simpler polynomials, such as $x^2 + 7x + 12 = (x + 3)(x + 4)$. Start with the greatest common factor, then use a pattern, trial or grouping, and check by multiplying. Setting each factor to zero gives the zeros of the polynomial.

Algebra 1 Common Core: A-SSE.A.2 · A-SSE.B.3a

Key facts:

- Always factor out the greatest common factor first.
- For $x^2 + bx + c$, find two numbers with product c and sum b .
- For $ax^2 + bx + c$, split the middle term using two numbers with product ac , then factor by grouping.
- Patterns: $a^2 - b^2 = (a + b)(a - b)$ and $a^2 \pm 2ab + b^2 = (a \pm b)^2$.
- The zero product property turns a factored equation into its zeros, the x-intercepts of the graph.

What does it mean to factor a polynomial?

To **factor** a polynomial is to write it as a product of simpler polynomials. Multiplying and factoring undo each other, so you can always check a factorization by expanding it. The first move is to pull out the **greatest common factor (GCF)**: $12x^3 + 18x^2 = 6x^2(2x + 3)$.

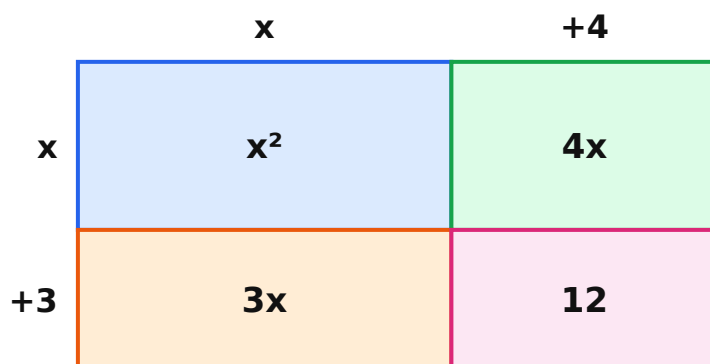
Vocabulary

Factor: an expression multiplied with others to make a product.

GCF: the largest number and highest power of each variable that divide every term.

Prime polynomial: a polynomial that cannot be factored into simpler polynomials with integer coefficients.

Zero product property: if $pq = 0$, then $p = 0$ or $q = 0$.



Factoring is multiplication run backward: a rectangle with area $x^2+7x+12$ has sides $x+4$ and $x+3$.

How do you factor $x^2 + bx + c$?

1. Find two integers whose product is c and whose sum is b .
2. Write the binomials $(x+m)(x+n)$.
3. Multiply to check.

Worked example. For $x^2+7x+12$ the numbers are 3 and 4, so it factors as $(x+3)(x+4)$. If c is negative, the two numbers have opposite signs: $x^2-2x-3=(x-3)(x+1)$.

factor pair of 12	product	sum
1 and 12	12	13
2 and 6	12	8
3 and 4	12	7 (match)

To factor $x^2+7x+12$, search for the pair with product 12 and sum 7.

How do you factor $ax^2 + bx + c$?

Multiply $a \cdot c$, find two numbers with that product and sum b , split the middle term, then factor by grouping. For $2x^2+7x+3$, $ac=6$ and the numbers are 1 and 6:

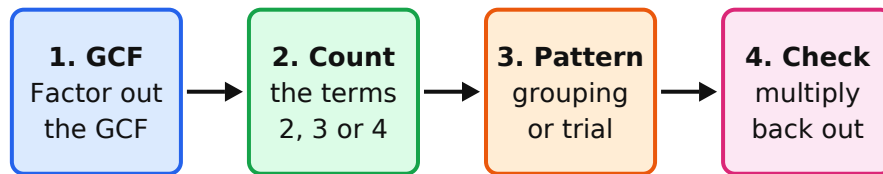
$$2x^2+x+6x+3 = x(2x+1)+3(2x+1) = (2x+1)(x+3).$$

What are the factoring patterns?

Pattern	Factored form
a^2-b^2	$(a+b)(a-b)$
$a^2+2ab+b^2$	$(a+b)^2$
$a^2-2ab+b^2$	$(a-b)^2$

A sum of squares such as x^2+9 is prime. With four terms, try grouping:

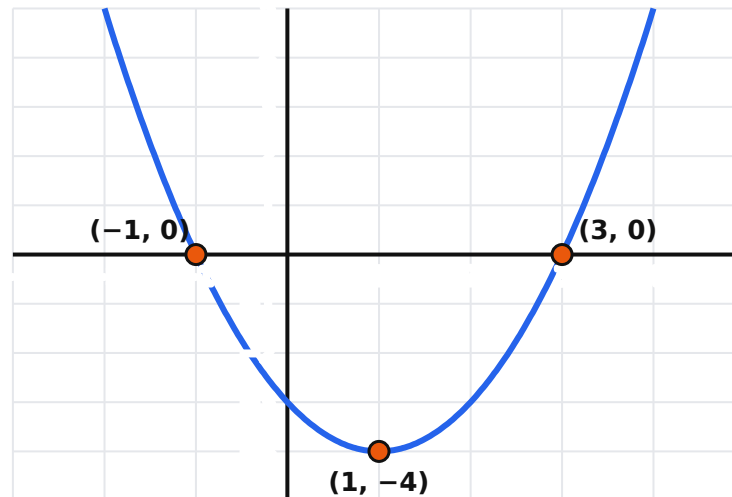
$$x^3+4x^2+3x+12 = x^2(x+4)+3(x+4) = (x^2+3)(x+4).$$



A reliable order of steps for factoring any polynomial.

How does factoring help find zeros?

Set the factored expression equal to zero and apply the zero product property. For $x^2-2x-3=0$, factor to get $(x-3)(x+1)=0$, so $x=3$ or $x=-1$. These zeros are the x-intercepts of the parabola.



The graph of $y = x^2-2x-3 = (x-3)(x+1)$ crosses the x-axis at the zeros $x=3$ and $x=-1$.

Factoring Polynomials: frequently asked questions

How do you know a polynomial is factored completely?

Every factor is prime and no common factor remains. For example, $4x^2-36 = (2x-6)(2x+6)$ is not complete because each binomial still has a factor of 2. The complete form is $4(x-3)(x+3)$.

What is the zero product property?

If a product equals zero, at least one factor equals zero. So $(x-5)(x+2)=0$ means $x=5$ or $x=-2$. It works only when the other side is exactly zero.

Why should you check a factorization by multiplying?

Expanding the factors must give back the original polynomial. This catches sign errors, such as writing $(x+4)(x-3)$ when the middle term needs $(x-4)(x+3)$.

Can every quadratic be factored?

Not with integer coefficients. The trinomial x^2+x+1 has no integer pair with product 1 and sum 1, so it is prime. Some quadratics still have zeros that are not integers.

Why can't $x^2 + 9$ be factored?

The difference of squares pattern needs a minus sign. A sum of squares has no real factors with integer coefficients, and its graph never touches the x-axis.