

Algebra 1 · Exponent Rules, Radicals and Rational Exponents · Lesson

Common Core N-RN.A.1, N-RN.A.2 · mathovore.fr

A rational exponent writes a root as a power: $x^{1/n}$ is the n th root of x , and $x^{m/n}$ means take the n th root, then raise to the m th power. All integer exponent rules still hold for these new exponents, so $8^{2/3} = 4$ and $\sqrt{72} = 6\sqrt{2}$.

Algebra 1 Common Core: N-RN.A.1 · N-RN.A.2

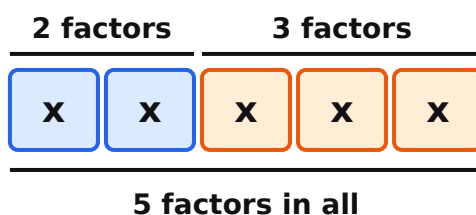
Key facts:

- Product rule: $x^m \cdot x^n = x^{m+n}$; quotient rule: subtract the exponents.
- Power of a power: multiply the exponents, $(x^m)^n = x^{mn}$.
- $x^0 = 1$ and $x^{-n} = \frac{1}{x^n}$ for $x \neq 0$.
- $x^{m/n} = (\sqrt[n]{x})^m$: root in the denominator, power in the numerator.
- Simplify a square root by pulling out the largest perfect square factor.

What are the rules for integer exponents?

An **exponent** tells how many times a base is used as a factor. In x^5 , the base is x and the exponent is 5. Every rule below comes from counting factors. Assume $x \neq 0$ and $y \neq 0$.

Rule	Pattern	Example
Product	$x^m \cdot x^n = x^{m+n}$	$x^3 \cdot x^4 = x^7$
Quotient	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^9}{x^5} = x^4$
Power of a power	$(x^m)^n = x^{mn}$	$(x^2)^5 = x^{10}$
Power of a product	$(xy)^n = x^n y^n$	$(3x)^2 = 9x^2$
Zero exponent	$x^0 = 1$	$7^0 = 1$
Negative exponent	$x^{-n} = \frac{1}{x^n}$	$2^{-3} = \frac{1}{8}$



Counting factors: $x^2 \cdot x^3$ has $2 + 3 = 5$ factors of x , so it equals x^5 .

Why is $x^0 = 1$? Look at the powers of 2 and keep dividing by 2. The pattern continues past 1 without a break.

n	3	2	1	0	-1	-2
2^n	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$

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Each step down divides by 2, so the pattern forces $2^0 = 1$, $2^{-1} = \frac{1}{2}$ and $2^{-2} = \frac{1}{4}$.

What does a rational exponent mean?

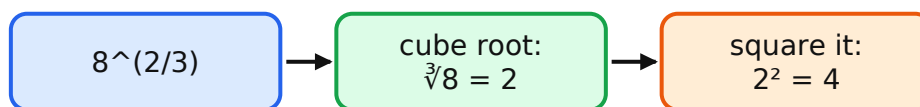
Rational exponent

$x^{1/n} = \sqrt[n]{x}$, the number whose n th power is x .

$x^{m/n} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$. The denominator is the root, the numerator is the power.

This definition keeps the power rules working: $(x^{1/2})^2 = x^1$, so $x^{1/2}$ must be a number that squares to x .

Example. $8^{2/3} = (\sqrt[3]{8})^2 = 2^2 = 4$. Take the root first; the numbers stay small.



$8^{2/3}$: the denominator 3 asks for a cube root, the numerator 2 asks for a square. The result is 4.

How do you simplify a square root?

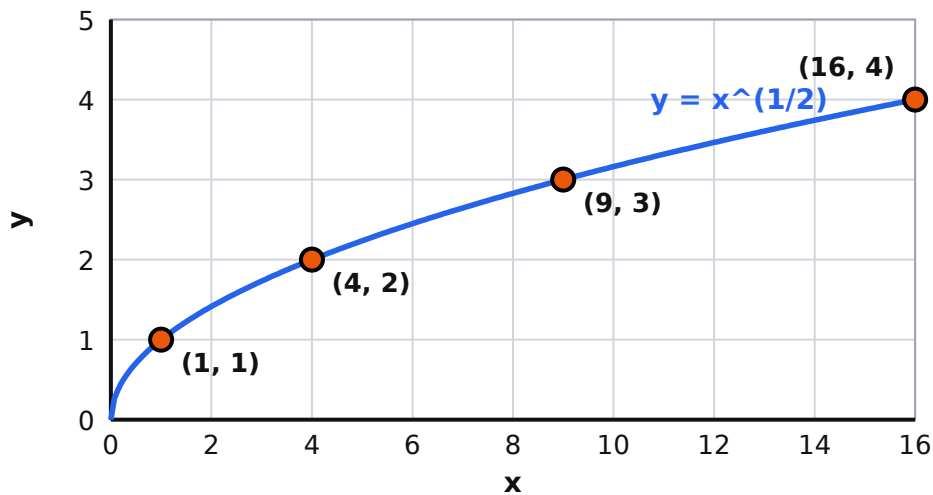
1. Find the largest perfect square that divides the number under the root.
2. Split the root: $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.
3. Replace the root of the perfect square with its value and keep what is left under the root.

Worked example. $\sqrt{72} = \sqrt{36 \cdot 2} = \sqrt{36} \cdot \sqrt{2} = 6\sqrt{2}$. Check: $6\sqrt{2} \approx 8.49$ and $\sqrt{72} \approx 8.49$.



Simplifying $\sqrt{72}$ by pulling out the largest perfect square, 36.

With variables that are positive, an even power comes out whole: $\sqrt{x^6} = x^3$. The graph of $y = x^{1/2}$ shows how slowly square roots grow.



The graph of $y = x^{1/2}$ passes through the points where x is a perfect square.

Exponent Rules, Radicals and Rational Exponents: frequently asked questions

What is a number to the zero power?

Any nonzero number to the zero power equals 1. The pattern $2^3 = 8$, $2^2 = 4$, $2^1 = 2$ divides by 2 each step, so the next value is $2^0 = 1$. The expression 0^0 is left undefined in Algebra 1.

What does a negative exponent mean?

A negative exponent means a reciprocal, not a negative number. For example, $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$. Move the base across the fraction bar and the exponent becomes positive.

How do you write a radical as a rational exponent?

Put the index of the root in the denominator and the power inside the root in the numerator. So $\sqrt[3]{x^5} = x^{5/3}$ and $\sqrt{y} = y^{1/2}$.

When do you add exponents and when do you multiply them?

Add exponents when multiplying powers with the same base: $x^2 \cdot x^3 = x^5$. Multiply exponents when raising a power to a power: $(x^2)^3 = x^6$.

How do you know a square root is fully simplified?

No perfect square larger than 1 divides the number under the root, and no fraction or root is left in a denominator. $5\sqrt{2}$ is simplified, but $\sqrt{50}$ is not because 25 divides 50.