

Algebra 1 · Exponential Growth and Decay · Lesson

Common Core F-LE.A.1a, F-LE.A.1c, F-LE.A.2, F-LE.A.3, F-IF.C.8b, F-LE.B.5 · mathovore.fr

An exponential function has the form $y = a \cdot b^x$: the starting value a is multiplied by the same factor b for every step in x . When $b > 1$ the quantity grows, and when $1 > b > 0$ it decays. Exponential growth eventually exceeds any linear growth, however large the line's slope.

Algebra 1 Common Core: F-LE.A.1a · F-LE.A.1c · F-LE.A.2 · F-LE.A.3 · F-IF.C.8b · F-LE.B.5

Key facts:

- Linear functions add a constant difference; exponential functions multiply by a constant factor.
- In $y = a \cdot b^x$, a is the value at $x = 0$ and b is the factor per step.
- Growth by r means $b = 1 + r$; decay by r means $b = 1 - r$.
- Compound interest: $A = P(1 + \frac{r}{n})^{nt}$.
- An exponential function eventually passes every linear function.

What is an exponential function?

An **exponential function** has the form $y = a \cdot b^x$, where a is the starting value (the output at $x = 0$) and b is the **growth or decay factor**. The factor b is positive and not equal to 1. The variable is in the exponent.

Linear or exponential?

Linear: equal steps in the input add equal amounts to the output (a constant difference).

Exponential: equal steps in the input multiply the output by equal factors (a constant ratio).

x	0	1	2	3	4
y	2	5	8	11	14



Linear: equal steps in x add the same amount, $+3$, every time.

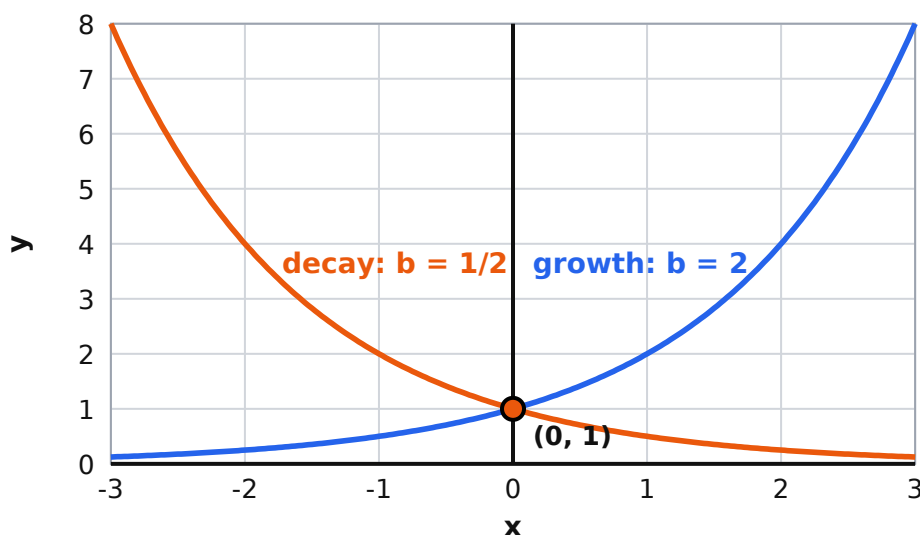
x	0	1	2	3	4
y	2	6	18	54	162



Exponential: equal steps in x multiply by the same factor, $\times 3$, every time.

How do percent rates become the factor b ?

A quantity that grows by r each period has $b = 1 + r$. A quantity that decays by r each period has $b = 1 - r$. A 5% increase gives $b = 1.05$. A 12% decrease gives $b = 0.88$. If $b > 1$ the graph rises; if $0 < b < 1$ it falls toward the x -axis.



Both graphs cross the y -axis at $(0, 1)$ and never touch the x -axis.

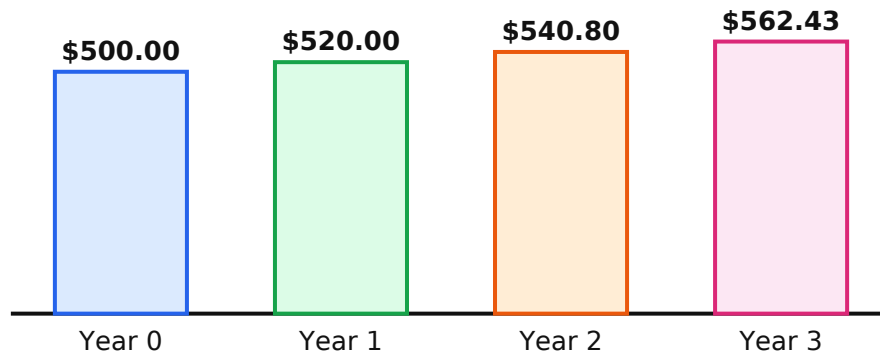
How does compound interest work?

Interest that is added to the balance also earns interest, so the balance is exponential:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$. Here P is the principal, r the yearly rate, n the compounds per year and t the

years.

Example. \$500 at 4% compounded yearly for 3 years: $A = 500(1.04)^3 \approx \$562.43$.



Balance of \$500 at 4% interest, compounded yearly. The bars grow by a bigger dollar amount each year.

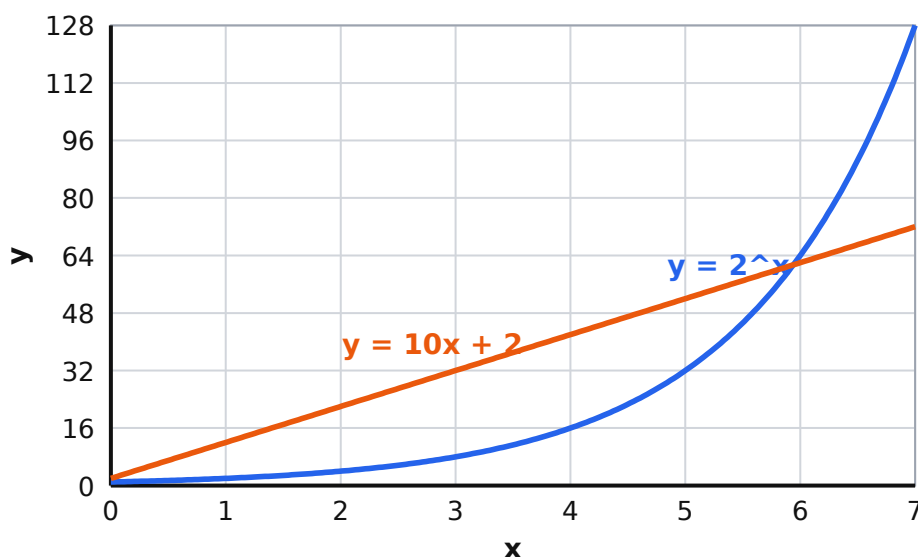
How do you write an exponential function from a description?

1. Find the starting value a .
2. Convert the percent rate to the factor b .
3. Write $y = a \cdot b^x$ and check one known value.

Worked example. A town has 2,400 people and grows 3% per year. Then $a = 2400$, $b = 1.03$ and $P(t) = 2400(1.03)^t$. After 5 years, $P(5) \approx 2,782$ people.

Does an exponential function always beat a linear one?

Yes, eventually. A line adds the same amount each step, but an exponential adds more each step. In the graph, $y = 2^x$ starts below $y = 10x + 2$, yet it passes the line before $x = 6$ because $2^6 = 64$ and $10(6) + 2 = 62$.



The line leads at first. After the curves cross, between $x=5$ and $x=6$, the exponential stays above.

Exponential Growth and Decay: frequently asked questions

How can you tell whether a table is linear or exponential?

Check equal steps in x . If the differences between outputs are equal, it is linear. If the ratios of consecutive outputs are equal, it is exponential. The outputs 3, 6, 12, 24 have a ratio of 2 each time.

What is the difference between a growth factor and a growth rate?

The rate is the percent change, such as 8%, written 0.08. The factor is what you multiply by, $1+0.08=1.08$. Decay works the same way: a 20% loss has a factor of 0.80.

Why can b not be 1 or negative in an exponential function?

If $b=1$, every output equals a , so the function is constant. A negative b would make the outputs flip between positive and negative, which does not model steady growth or decay.

What does the number a mean in $y = a \cdot b^x$?

It is the initial value, the output when $x=0$, because $b^0=1$. In $y=250(0.9)^x$, the quantity starts at 250 and loses 10% each step.

Will an exponential function always exceed a linear function?

Eventually, yes. A line may stay ahead for a while, but each exponential step is larger than the last. For $y=2^x$ and $y=10x+2$, the exponential passes the line at $x=6$.