

Algebra 1 · Building Functions and Inverses · Worksheet Answer Key

Common Core F-BF.A.1a, F-BF.A.1b, F-BF.B.4a, F-LE.A.1b · mathovore.fr

Every solution shows the setup, the arithmetic and a check, with the final answer in bold. Compare your steps with these, especially the order of operations when you write a function and the minus sign when you subtract a cost function.

Algebra 1 Common Core: F-BF.A.1a · F-BF.A.1b · F-BF.B.4a · F-LE.A.1b

Problem 1: Steps to a function

Multiplying by 4 gives $4x$ and subtracting 7 gives $f(x) = 4x - 7$.

$$f(5) = 4(5) - 7 = 20 - 7 = 13. \quad f(x) = 4x - 7, \quad f(5) = 13$$

Problem 2: Recursive growth

Start: $h(0) = 6$. Each week adds 2: $h(n) = h(n-1) + 2$.

$$h(3) = 10 + 2 = 12 \text{ and } h(4) = 12 + 2 = 14. \quad h(4) = 14 \text{ inches}$$

Problem 3: Recursive to explicit

The value starts at 5 and 3 is added n times, so $f(n) = 3n + 5$.

$$f(10) = 3(10) + 5 = 35. \quad f(n) = 3n + 5, \quad f(10) = 35$$

Problem 4: Order of the steps

Add 6 first, so the sum must be in parentheses: $g(x) = 3(x + 6)$.

$$g(4) = 3(4 + 6) = 3(10) = 30. \quad g(x) = 3(x + 6), \quad g(4) = 30$$

Problem 5: Taxi fare function

Cost is the flat fee plus the rate times the miles: $c(m) = 2.25m + 3$.

$$c(8) = 2.25(8) + 3 = 18 + 3 = 21. \quad c(m) = 2.25m + 3; \text{ the ride costs } \mathbf{\$21.00}$$

Problem 6: Sum of two functions

$$(f+g)(x) = (3x+2) + (x-5) = 4x-3.$$

$$(f+g)(2) = 4(2)-3 = 5. \text{ Check: } f(2) = 8 \text{ and } g(2) = -3, \text{ and } 8+(-3) = 5. 4x-3; 5$$

Problem 7: Difference of two functions

$$(f-g)(x) = (5x-1) - (2x+8) = 5x-1-2x-8 = 3x-9.$$

$$(f-g)(6) = 3(6)-9 = 9. \text{ Check: } f(6) = 29, g(6) = 20, \text{ and } 29-20 = 9. 3x-9; 9$$

Problem 8: Product of two functions

$$(f \cdot g)(x) = (x+3)(2x) = 2x^2+6x.$$

$$(f \cdot g)(4) = 2(16)+6(4) = 32+24 = 56. \text{ Check: } f(4) \cdot g(4) = 7 \cdot 8 = 56. 2x^2+6x; 56$$

Problem 9: Profit function

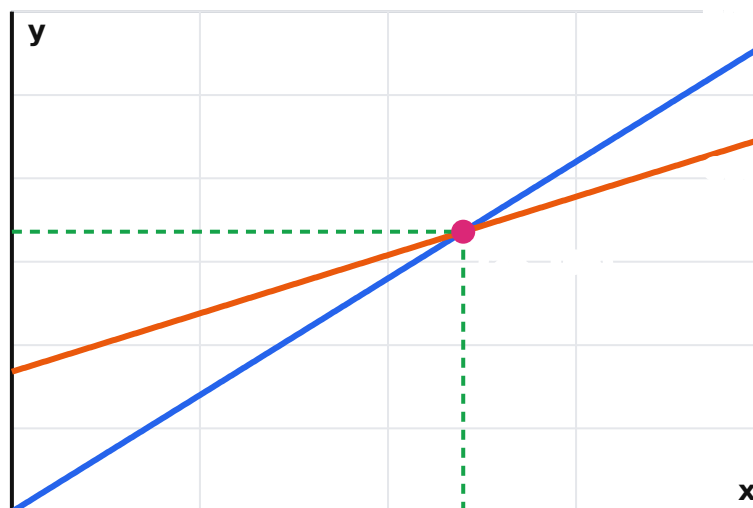
$$P(x) = R(x) - C(x) = 6x - (2x+40) = 4x-40.$$

$$P(25) = 4(25)-40 = 60. \text{ Check: } R(25) = 150, C(25) = 90, 150-90 = 60. P(x) = 4x-40; \text{ profit } \$60$$

Problem 10: Break-even from a graph

Break even when revenue equals cost: $7x = 3.5x + 84$, so $3.5x = 84$ and $x = 24$.

$R(24) = 168$ and $C(24) = 3.5(24) + 84 = 84 + 84 = 168$. **24 hats, revenue \$168**



The lines cross at (24, 168).

Problem 11: Solve $f(x) = c$

$3x-4=20$, so $3x=24$ and $x=8$. Check: $f(8)=24-4=20$. $x=8$

Problem 12: Inverse of a linear function

$y=5x+10$, so $y-10=5x$ and $x=\frac{y-10}{5}=\frac{y}{5}-2$.

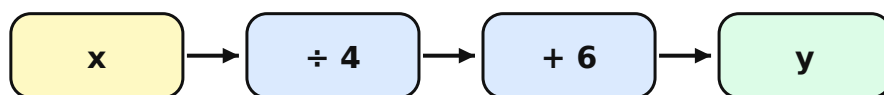
Check: $f(3)=25$ and $\frac{25-10}{5}=3$. $x=\frac{y-10}{5}$

Problem 13: Inverse with a fraction

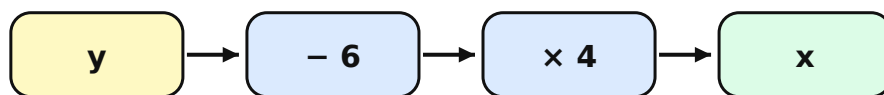
$y=\frac{x}{4}+6$, so $y-6=\frac{x}{4}$ and $x=4(y-6)=4y-24$. The inverse undoes "add 6" first, then "divide by 4".

For $y=9$: $x=4(9-6)=12$. Check: $f(12)=3+6=9$. $x=4y-24$; **input 12**

f: forward



inverse: undo in reverse order



Forward and inverse machines for this function.

Problem 14: Check an inverse

$f(5)=8$ and $g(8)=\frac{15}{3}=5$. Also $g(5)=\frac{12}{3}=4$ and $f(4)=12-7=5$.

Each function returns the original input, so **g is the inverse of f** .

Problem 15: Is the rate constant?

x rises by 2 each step and y rises by 6 each step ($11-5=6$, $17-11=6$, $23-17=6$), so the rate is constant: $\frac{6}{2}=3$ per 1 unit.

The start value at $x=0$ is 5, so $f(x)=3x+5$. Check: $f(6)=23$. **constant rate 3**; $f(x)=3x+5$

		$+2$	$+2$	$+2$
x	0	2	4	6
y	5	11	17	23
		$+6$	$+6$	$+6$

Equal steps in x give equal changes in y.

Problem 16: Compare two tables

Table A: $12-7=5$, $17-12=5$, $22-17=5$, always $+5$ for each 1 step in x .

Table B: $6-3=3$, $12-6=6$, $24-12=12$, the changes keep doubling. **Only Table A has a constant rate (5 per unit)**

		$+1$	$+1$	$+1$
Table A: x	1	2	3	4
y	7	12	17	22
		$+5$	$+5$	$+5$

Table A: constant $+5$.

Problem 17: Temperature conversion

(a) $1.8c+32=68$, so $1.8c=36$ and $c=20$.

(b) $c = \frac{F-32}{1.8}$. Check with 95°F : $\frac{95-32}{1.8} = \frac{63}{1.8} = 35$ and $1.8(35)+32=95$. **20°C** ; $c = \frac{F-32}{1.8}$

Problem 18: Play ticket profit

$$R(x)=8x, C(x)=1.25x+150, P(x)=8x-(1.25x+150)=6.75x-150.$$

A profit needs $6.75x > 150$, so x is greater than 22.2. Test the whole numbers 22 and 23:

$$P(22)=148.5-150=-1.5 \text{ (a loss) and } P(23)=155.25-150=5.25. \text{ **23 tickets** }$$

Problem 19: Taxi fare inverse

$$c-3=2.25m, \text{ so } m = \frac{c-3}{2.25}.$$

$$m = \frac{25.5-3}{2.25} = \frac{22.5}{2.25} = 10. \text{ Check: } 2.25(10)+3=25.5. \text{ **10 miles** }$$

Problem 20: Savings recursion

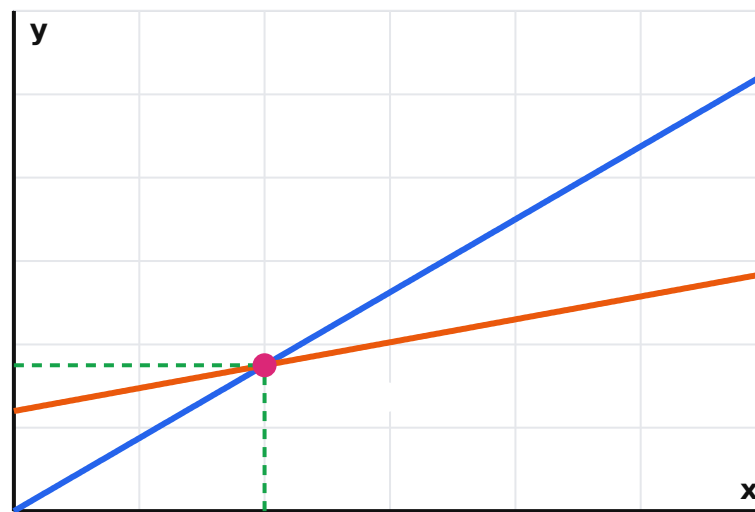
Recursive: $S(0) = 45$, $S(n) = S(n-1) + 12$. Explicit: $S(n) = 12n + 45$.

$12n + 45 = 165$, so $12n = 120$ and $n = 10$. Check: $12(10) + 45 = 165$. **10 weeks**

Problem 21: Hot dog stand profit

$$P(x) = 3.5x - (1.1x + 48) = 2.4x - 48.$$

Break even: $2.4x = 48$, so $x = 20$. Profit for 50: $P(50) = 2.4(50) - 48 = 120 - 48 = 72$. **20 hot dogs; profit \$72**



Break-even at (20, 70).

Problem 22: Gas tank inverse

$$0.04m = 14 - g, \text{ so } m = \frac{14-g}{0.04} = 350 - 25g.$$

For $g = 6$: $m = 350 - 150 = 200$. For $g = 0$: $m = 350$. Check: $14 - 0.04(200) = 6$. **200 miles; 350 miles**

Problem 23: Two classes combined

(a) $T(x) = (4x - 20) + (6x - 35) = 10x - 55$.

(b) $10x - 55 = 145$, so $10x = 200$ and $x = 20$. Check: $L(20) = 60$, $C(20) = 85$, $60 + 85 = 145$.

(c) $x = \frac{T+55}{10}$. $10x - 55$; **20 hours**; $x = \frac{T+55}{10}$