

Algebra 1 · Building Functions and Inverses · Lesson

Common Core F-BF.A.1a, F-BF.A.1b, F-BF.B.4a, F-LE.A.1b · mathovore.fr

To build a function, name the input and the output, then write how the output is produced: explicitly, like $f(h) = 8h + 12$, or recursively, as a start value plus a repeated change. Functions combine by adding, subtracting or multiplying, and a linear function is undone by solving $f(x) = c$ for x .

Algebra 1 Common Core: F-BF.A.1a · F-BF.A.1b · F-BF.B.4a · F-LE.A.1b

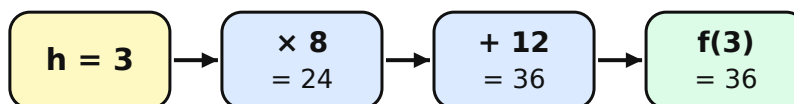
Key facts:

- Standards F-BF.A.1a, F-BF.A.1b, F-BF.B.4a and F-LE.A.1b: write functions from contexts, combine them, find simple inverses and spot constant rates.
- Explicit $f(h) = 8h + 12$ and recursive $f(0) = 12$, $f(h) = f(h-1) + 8$ describe the same situation.
- Profit equals revenue minus cost: $P(x) = R(x) - C(x)$; break-even is where $P(x) = 0$.
- To invert $y = 2x + 3$, solve for x : $x = \frac{y-3}{2}$.
- A constant rate of change means equal input steps give equal output changes.

How do you build a function from a situation?

Name the input and the output, then describe how the output is produced. The rule can be an **explicit expression**, a **recursive process** (a starting value plus a repeated change), or a list of **calculation steps**. Jamal rents a kayak for a \$12 fee plus \$8 per hour. The steps are: multiply the hours by 8, then add 12.

Explicit: $f(h) = 8h + 12$. Recursive: $f(0) = 12$ and $f(h) = f(h-1) + 8$. Both give $f(3) = 36$, so 3 hours cost \$36.



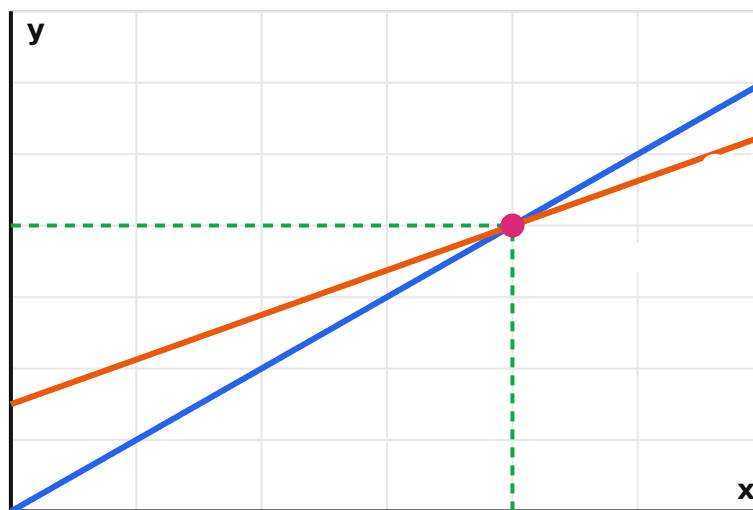
Calculation steps for the kayak rental with 3 hours: multiply by 8, then add 12.

How do you combine functions with arithmetic?

Combining functions

$(f+g)(x) = f(x) + g(x)$, $(f-g)(x) = f(x) - g(x)$, $(f \cdot g)(x) = f(x) \cdot g(x)$. Profit is revenue minus cost:
 $P(x) = R(x) - C(x)$.

Maya sells muffin boxes for \$4 each, so $R(x) = 4x$. Supplies cost \$30 plus \$2.50 per box, so $C(x) = 2.5x + 30$. Then $P(x) = 4x - (2.5x + 30) = 1.5x - 30$. The break-even point is where $P(x) = 0$, so $x = 20$ boxes, where revenue and cost are both \$80.



Revenue and cost lines cross at the break-even point (20, 80).

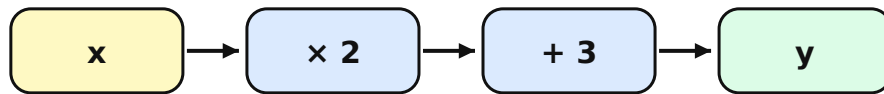
How do you find the inverse of a linear function?

Solving $f(x) = c$ for x undoes the function. Follow these steps:

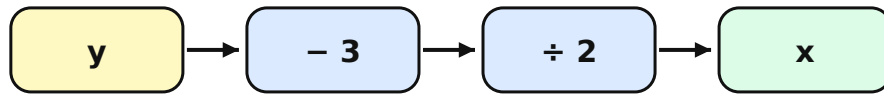
1. Write $y = f(x)$.
2. Undo the operations in reverse order, starting with the last one applied.
3. The expression you get for x in terms of y is the inverse.
4. Check by substituting a value.

Example: $y = 2x + 3$ gives $y - 3 = 2x$, so $x = \frac{y-3}{2}$. If $f(x) = 11$, then $x = \frac{11-3}{2} = 4$, and indeed $f(4) = 11$.

f: forward

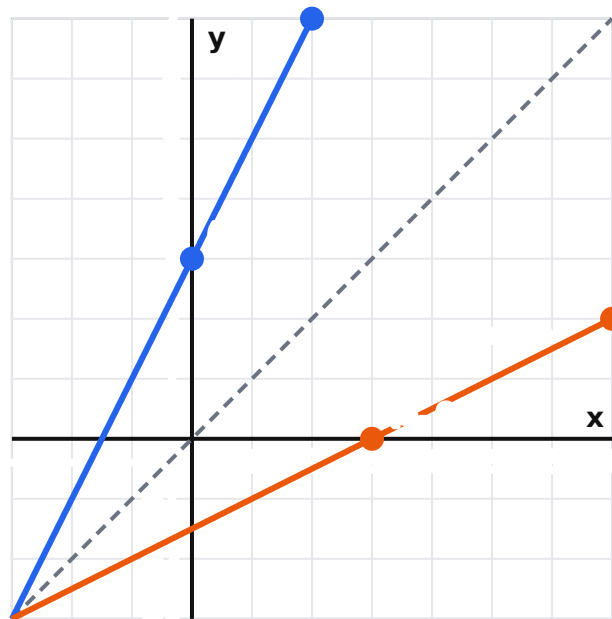


inverse: undo in reverse order



The inverse reverses the order of the operations and undoes each one.

A point (a,b) on the graph of f matches the point (b,a) on the inverse, so the two graphs mirror each other across the line $y = x$.



The graph of f and its inverse are reflections across the dashed line $y = x$.

What does a constant rate of change look like in a table?

A quantity changes at a **constant rate per unit interval** when equal steps in x always produce equal changes in the output. In the kayak table the cost rises by \$8 for every extra hour, so the situation is linear and the slope is 8. If the differences change from step to step, the function is not linear.

		+1	+1	+1	+1
Hours h	0	1	2	3	4
Cost	12	20	28	36	44
		+8	+8	+8	+8

Every 1-hour step adds exactly \$8, a constant rate of change.

Building Functions and Inverses: frequently asked questions

What is the difference between an explicit and a recursive function?

An explicit rule computes any output straight from the input, such as $f(h) = 8h + 12$. A recursive rule gives a starting value and tells how to get each output from the previous one. Both can describe the same situation.

How do you write a profit function?

Write the revenue function and the cost function, then subtract: $P(x) = R(x) - C(x)$. Distribute the minus sign over every term of the cost, including the fixed fee.

What is the inverse of a linear function?

It is the function that reverses the original. To find it, solve $y = f(x)$ for x . Its graph is the reflection of the graph of f across the line $y = x$.

How can you tell from a table that the rate is constant?

Compare the change in the output with the change in the input for each pair of neighboring rows. If every ratio of changes is the same number, the rate of change is constant and the function is linear.

Does every function have an inverse?

No. A linear function with slope 0 gives the same output for every input, so you cannot tell which input was used. Every linear function with a nonzero slope has an inverse that is also linear.